INTRODUCTION TO NUCLEAR REACTOR PHYSICS

The purpose of this Document on **Nuclear Reactor Physics** is the non-technical introduction of some of the most important concepts of physics required for a general understanding of the generation of energy through the use of nuclear power.

Inherent in any discussion on the structure of matter and certainly in the discussion of the production of electrical energy through the fission of nuclear material are some of the topics of modern physics: **Quantum Mechanics and Special Relativity.**

In this introduction to nuclear physics we shall consider only those laws of mechanics, thermodynamics and hydraulics which can appropriately be discussed in the guise of "Rational Mechanics", and we will use the concepts which form the basis of modern Physics: **Quantum Mechanics and Special Relativity** only when absolutely necessary to impart the information needed to the reader to continue the study in this white paper.

The discovery of fission in 1939 was and event of epochal significance in the annals of physics because it ushered in the age of the atom. This discovery opened up the prospect of an entirely new source of power utilizing the internal binding energy of the atom.

The operation of a nuclear reactor depends upon various interactions of neutrons with atomic nuclei. In order to appreciate the complexities of a nuclear reactor it is desirable to consider briefly some of the fundamental of atomic and nuclear physics. This paper was produced to provide such an introduction.

Live Blessed!

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CHAPTER ONE: INTRODUCTION TO NUCLEAR PHYSICS

SECTION ONE: BASIC CONCEPTS

1.1 History of Structure of Matter

Early Greek philosophers speculated that the earth was made up of different combinations of basic substances, or elements. They considered these basic elements to be earth, air, water, and fire. Modern science shows that the early Greeks held the correct concept that matter consists of a combination of basic elements, but they incorrectly identified the elements.

In 1661 the English chemist Robert Boyle published the modern criterion for an element. He defined an element to be a basic substance that cannot be broken down into any simpler substance after it is isolated from a compound, but can be combined with other elements to form compounds. To date, 105 different elements have been confirmed to exist, and researchers claim to have discovered three additional elements. Of the 105 confirmed elements, 90 exist in nature and 15 are man-made.

Another basic concept of matter that the Greeks debated was whether matter was continuous or discrete. That is, whether matter could be continuously divided and subdivided into ever smaller particles or whether eventually an indivisible particle would be encountered. Democritus in about 450 B.C. argued that substances were ultimately composed of small, indivisible particles that he labeled "**atoma**". He further suggested that different substances were composed of different atoms or combinations of atoms, and that one substance could be converted into another by rearranging the atoms. It was impossible to conclusively prove or disprove this proposal for more than 2000 years.

The modern proof for the atomic nature of matter was first proposed by the English chemist John Dalton in 1803. Dalton stated that each chemical element possesses a particular kind of atom, and any quantity of the element is made up of identical atoms of this kind. What distinguishes one element from another element is the kind of atom of which it consists, and the basic physical difference between kinds of atoms is their weight.

1.2 Subatomic Particles

For almost 100 years after Dalton established the atomic nature of atoms, it was considered impossible to divide the atom into even smaller parts. All of the results of chemical experiments during this time indicated that the atom was indivisible. Eventually, experimentation into electricity and radioactivity indicated that particles of matter smaller than the atom did indeed exist.

In 1906, J. J. Thompson won the Nobel Prize in physics for establishing the existence of electrons. **Electrons** are negatively-charged particles that have 1/1835 the mass of the hydrogen atom. Soon after the discovery of electrons, protons were discovered. **Protons** are relatively large particles that have

almost the same mass as a hydrogen atom and a positive charge equal in magnitude (but opposite in sign) to that of the electron. The third subatomic particle to be discovered, the **neutron**, was not found until 1932. The **neutron** has almost the same mass as the proton, but it is electrically neutral.

1.3 Bohr Model of the Atom

The British physicist Ernest Rutherford postulated that the positive charge in an atom is concentrated in a small region called a nucleus at the center of the atom with electrons existing in orbits around it.

Niels Bohr, coupling Rutherford's postulation with the newly minted theories of quantum mechanics introduced by Max Planck, proposed that the atom consists of a dense nucleus of **protons** surrounded by **electrons** traveling in discrete orbits at fixed distances from the nucleus.

An electron in one of these stationary orbits or shells has a specific or discrete quantity of energy (quantum). When an electron moves from one allowed orbit to another allowed orbit, the energy difference between the two states is emitted or absorbed in the form of a single quantum of radiant energy called a **photon**.

The Quantum of energy emitted from this jump from one stationary state to another is given by the Plank formula:

$$\mathbf{E = h\nu}$$

Where **h** = Planck's constant = 6.63×10^{-34} J-s

ν= frequency of the photon.

Bohr's theory was the first to successfully account for the discrete energy levels of this radiation as measured in the laboratory. Although Bohr's atomic model was designed specifically to explain the hydrogen atom, his theories apply generally to the structure of all atoms. Additional information on electron shell theory can be found in any introductory book on **Quantum Mechanics.**

1.4 Measuring Units on the Atomic Scale

The size and mass of atoms are so small that the use of normal measuring units, while possible, is often inconvenient. Units of measure have been defined for mass and energy on the atomic scale to make measurements more convenient to express.

The unit of measure for mass is the **atomic mass unit** (**amu**). One atomic mass unit is equal to 1.66×10^{-24} grams. The reason for this particular value for the atomic mass unit will be made clear later in this introduction. Note that the mass of a neutron and a proton are both about 1 amu.

The unit for energy is the **electron volt** (**eV**). The electron volt is the amount of energy acquired by a single electron when it falls through a potential difference of one volt. One electron volt is equivalent to 1.602×10^{-19} joules or 1.18×10^{-19} foot-pounds.

1.5 Nuclides

The total number of protons in the nucleus of an atom is called the ***atomic number*** of the atom and is given the symbol **Z**. The number of electrons in an electrically-neutral atom is the same as the number of protons in the nucleus.

The number of neutrons in a nucleus is known as the neutron number and is given the symbol **N**. The **mass number** of the nucleus is the total number of nucleons, that is, protons and neutrons in the nucleus. The mass number is given the symbol **A** and can be found by the equation

$$A = Z + N.$$

Each of the chemical elements has a unique atomic number because the atoms of different elements contain a different number of protons. The atomic number of an atom identifies the particular element.

Each type of atom that contains a unique combination of protons and neutrons is called a **nuclide.** Not all combinations of numbers of protons and neutrons are possible, but about 2500 specific nuclides with unique combinations of neutrons and protons have been identified. Each nuclide is denoted by the chemical symbol of the element with the atomic number written as a subscript and the mass number written as a superscript.

Because each element has a unique name, chemical symbol, and atomic number, only one of the three is necessary to identify the element. For this reason nuclides can also be identified by either the chemical name or the chemical symbol followed by the mass number (for example, U-235 or uranium-235).

Another common format is to use the abbreviation of the chemical element with the mass number superscripted (for example, $\mathbf{U^{235}}$). In this white paper the format used will usually be the element's name followed by the mass number as a superscript.

1.6 Isotopes

Isotopes are nuclides that have the same atomic number and are therefore the same element, but differ in the number of neutrons. Most elements have a few stable isotopes and several unstable, radioactive isotopes. For example, oxygen has three stable isotopes that can be found in nature (**oxygen^{16}, oxygen^{17}**, **oxygen^{18},** and eight radioactive isotopes. Another example is hydrogen, which has two stable isotopes (**hydrogen-1**, **hydrogen^{1}** and hydrogen-2) and a single radioactive isotope (hydrogen-3, **hydrogen^{3}**).

The isotopes of hydrogen are unique in that they are each commonly referred to by a unique name instead of the common chemical element name. Hydrogen-1 is almost always referred to as hydrogen, but the term protium is infrequently used also. Hydrogen-2 is commonly called deuterium and Hydrogen-3 is commonly called tritium.

1.7 Atomic and Nuclear Radii

The size of an atom is difficult to define exactly due to the fact that the electron cloud, formed by the electrons moving in their various orbitals, does not have a distinct outer edge. A reasonable measure of atomic size is given by the average distance of the outermost electron from the nucleus.

Except for a few of the lightest atoms, the average atomic radii are approximately the same for all atoms, about **2×10^{-8} cm**. Like the atom the nucleus does not have a sharp outer boundary. Experiments have shown that the nucleus is shaped like a sphere with a radius that depends on the atomic mass number of the atom.

1.8 Nuclear Forces

In the Bohr model of the atom, the nucleus consists of positively-charged protons and electrically neutral neutrons. Since both protons and neutrons exist in the nucleus, they are both referred to as nucleons. One problem that the Bohr model of the atom presented was accounting for an attractive force to overcome the repulsive force between protons.

Two of the four forces present in the nucleus are;

1. Electrostatic forces between charged particles
2. Gravitational forces between any two objects that have mass.

It is possible to calculate the magnitude of the gravitational force and electrostatic force based upon principles from classical physics. Newton stated that the **gravitational force** between two bodies is directly proportional to the masses of the two bodies and inversely proportional to the square of the distance between the bodies. This relationship is shown in the equation below.

$$F_g = (G \times m_1 \times m_2)/ r_{12}$$

where:

F_g = gravitational force (newtons)
m_1 = mass of first body (kilograms)
m_2 = mass of second body (kilograms)
G = gravitational constant (6.67 x 10 -11 N-m2/kg2)
r = distance between particles (meters)

The equation illustrates that the larger the masses of the objects or the smaller the distance between the objects, the greater the gravitational force. So even though the masses of nucleons are very small, the fact that the distance between nucleons is extremely short may make the gravitational force significant. It is necessary to calculate the value for the gravitational force and compare it to the value for other forces to determine the significance of the gravitational force in the nucleus. The gravitational force between two protons that are separated by a distance of **10^{-20}** meters is about **10^{-24}** newtons.

Coulomb's Law can be used to calculate the force between two protons. The **electrostatic force** is directly proportional to the electrical charges of the two particles and inversely proportional to the square of the distance between the particles.

Coulomb's Law is stated as the following equation:

$$F_e = \frac{(K \times Q_1 \times Q_2)}{r_{12}}$$

where:

F_e = Electrostatic force (newtons)
K = electrostatic constant (9.0 x 109 N-m2/C2)
Q_1 = charge of first particle (coulombs)
Q_2 = charge of second particle (coulombs)
r_{12} = Distance between particles (meters)

Using this equation, the electrostatic force between two protons that are separated by a distance of **10^{-20}** meters is about **10^{12}** newtons. Comparing this result with the calculation of the gravitational force (**10^{-24}** newtons) shows that the gravitational force is so small that it can be neglected.

If only the electrostatic and gravitational forces existed in the nucleus, then it would be impossible to have stable nuclei composed of protons and neutrons. The gravitational forces are much too small to hold the nucleons together compared to the electrostatic forces repelling the protons. Since stable atoms of neutrons and protons do exist, there must be another attractive force acting within the nucleus. This force is called the **nuclear force**.

The **nuclear force** is a strong attractive force that is independent of charge. It acts equally only between pairs of neutrons, pairs of protons, or a neutron and a proton. The nuclear force has a very short range; it acts only over distances approximately equal to the diameter of the nucleus (**10^{-13}** cm). The attractive nuclear force between all nucleons drops off with distance much faster than the repulsive electrostatic force between protons.

1.9 Atomic Nature of Matter Summary

Structure of the Atom

An atom consists of a positively charged **Nucleus** surrounded by a number of negatively charged particles, called **electrons**, so that the atom as a whole is electrically neutral. The atomic nuclei are built up of two kinds of primary particles, namely **protons** and **neutrons**, which can be ordinarily referred to as **Nucleons.** The masses of the protons and neutrons are similar and much heavier than an electron, by a factor of 1840. As the nucleus contains all the protons and neutrons, it follows that the mass of the atoms are concentrated in the nucleus.

The proton carries a single unit of charge and the neutron is electrically neutral. The unit of charge carried by the proton is equal in magnitude and opposite in sign, to the charge on the electron. This charge is often referred to in physics as the fundamental charge.

Each electron carries a unit of negative charge equal to the charge on the proton. The number of orbital electrons is equal to the number of protons in the nucleus so that the their charge balances and overall the atom is electrically neutral. If the atom loses or gains a planetary electron, it is left with a residual electrical charge and the atom is said to be ionized.

The number of protons in the nucleus determines the number of orbital electrons. It is the electronic structure of the atom, in particular the outermost orbiting electrons, that give the atoms its chemical properties. Chemical reactions are due to electronic interactions.

In this paper we will only mention that the electron, in a stable orbit around the nucleus occupies a stationary state, often referred to in Physics as an Eigenstate or an Eigenfunction of the orbital decomposition of the central field problem. The solution of the electronic motion of an electron in a central field of force will not be addressed here. The force which holds the electron to the nucleus is the electrostatic force of electromagnetic theory and is accurately describe by an inverse square law of force, similar the Newton's law of gravitation.

SECTION TWO: STRUCTURE OF THE NUCLEUS

2.1. Atomic Number and Chart of Nuclides

As has been previously stated the nucleus consists of protons and neutrons. For a given element, the number of protons present in the atomic nucleus, which is the same as the number of positive charges it carries, is called the **atomic number.** This number is identical with the ordinal number of the element which is used in the familiar Periodic table of the elements. Thus the atomic number of hydrogen is 1, of helium 2, and lithium 3, up to 92 for **Uranium,** the element of highest atomic number existing in nature to any extent. A large number of heavier elements have been produced artificially, of these elements, plutonium, atomic number 94, is the most important because of it's connection with nuclear weapons.

The total number of **Nucleons,** i.e. of protons and neutrons, in an atomic nucleus is referred to as the **mass number.** Since the masses of neutrons and protons and very nearly identical, it is evident that the mass number is the integer nearest to the atomic weight of the species under considerations.

It is the atomic number, i.e. the number of protons in the nucleus, which determines the chemical nature of an element. This is because the chemical properties depend on the (orbital) electrons, surrounding the nucleus, and

their number must be equal to the number of protons in the nucleus since the atom must be electrically neutral. Consequently, atoms with nuclei containing the same numbers of protons, i.e. with the same atomic number, but with different numbers of neutrons, i.e. with different mass number, are essentially identical chemically. Such species having the same atomic number but different mass numbers are called **Isotopes.** Isotopes are, in general, chemically identical but have different atomic weight. They are, in general, indistinguishable chemically, but have different atomic weights. As of January 1, 1962, all atomic weights are expressed on a single scale which assigns a value of 12 to the common isotope of $\mathbf{C^{12}}$.

In Nuclear physics, and related fields, the masses of atoms, of nuclei, and of nuclear particles are invariably expressed on the so-called physical scale. The **Atomic Mass unit** (amu) is then defined as exactly one-twelfth of the mass of the $\mathbf{C^{12}}$ atom.

2.2 Chart of the Nuclides

A tabulated chart called the **Chart of the Nuclides** lists the stable and unstable nuclides in addition to pertinent information about each one. This chart is available from Wikipedia using the following URL: https://wikipedia.org/wiki/Table_of_nuclides. This chart plots a box for each individual nuclide, with the number of protons (**Z**) on the vertical axis and the number of neutrons (**N = A - Z**) on the horizontal axis. The chart indicates stable isotopes.

Some **isotopes** are **artificially radioactive**, meaning that they are produced by artificial techniques and do not occur naturally.

A. Information for Stable Nuclides

For the stable isotopes, in addition to the symbol and the atomic mass number, the number percentage of each isotope in the naturally occurring element is listed, as well as the thermal neutron activation cross section and the mass in atomic mass units (amu).

B. Information for Unstable Nuclides

For unstable isotopes the additional information includes the half life, the mode of decay (for example, b-, a), the total disintegration energy in **MeV** (**million electron volts**), and the mass in **amu** when available

C. Neutron - Proton Ratios

If you plot the number of protons on the x-axis and the number of neutrons on the y axis, then you will see that as the mass numbers become higher, the ratio of neutrons to protons in the nucleus becomes larger. For helium-4 (2 protons and 2 neutrons) and oxygen-16 (8 protons and 8 neutrons) this ratio is unity. For indium-115 (49 protons and 66 neutrons) the ratio of neutrons to protons has increased to 1.35, and for uranium-238 (92 protons and 146 neutrons) the neutron to-proton ratio is 1.59.

D. Natural Abundance of Isotopes

The relative abundance of an isotope in nature compared to other isotopes of the same element is relatively constant. The Chart of the Nuclides presents the relative abundance of the naturally occurring isotopes of an element in units of **atom percent**. **Atom percent** is the percentage of the atoms of an element that are of a particular isotope.

Atom percent is abbreviated as **a/o.** For example, if a cup of water contains 8.23×10^{24} atoms of oxygen, and the isotopic abundance of oxygen-18 is 0.20%, then there are 1.65×10^{22} atoms of oxygen-18 in the cup.

E. Atomic Weight

The **atomic weight** for an element is defined as the average atomic weight of the isotopes of the element. The atomic weight for an element can be calculated by summing the products of the isotopic abundance of the isotope with the atomic mass of the isotope.

SECTION THREE: MASS DEFECT AND BINDING ENERGY

3.1 MASS DEFECT

Careful measurements have shown that the mass of a particular atom is always slightly less than the sum of the masses of the individual neutrons, protons, and electrons of which the atom consists. The difference between the mass of the atom and the sum of the masses of its parts is called the **mass defect (D_m)**.

The mass defect can be calculated using the equation show below. In calculating the mass defect it is important to use the full accuracy of mass measurements because the difference in mass is small compared to the mass of the atom. Rounding off the masses of atoms and particles to three or four significant digits prior to the calculation will result in a calculated mass defect of zero.

$$D_m = [\, Z(m_p + m_e) + (A-Z)m_n \,] - m_{atom}$$

where:

D_m = mass defect (**amu**)
m_p = mass of a proton (1.007277 **amu**)
m_n = mass of a neutron (1.008665 **amu**)
m_e = mass of an electron (0.000548597 **amu**)
m_{atom} = mass of nuclide (**amu**)
Z = atomic number (number of protons)
A = mass number (number of nucleons)

3.2 Binding Energy

The loss in mass, or mass defect, is due to the conversion of mass to binding energy when the nucleus is formed. **Binding energy** is defined as the amount of energy that must be supplied to a nucleus to completely

separate its nuclear particles (nucleons). It can also be understood as the amount of energy that would be released if the nucleus was formed from the separate particles.

Binding energy is the energy equivalent of the mass defect. Since the mass defect was converted to binding energy (BE) when the nucleus was formed, it is possible to calculate the binding energy using a conversion factor derived by the mass-energy relationship from **Einstein's Theory of Relativity.**

Einstein's famous equation relating mass and energy is;

$$\mathbf{E = mc^2}$$

Where;

E = Energy in Joules
m = mass in kilograms
c = is the velocity of light (**c = 3 x 10^8 meters/sec**).

The energy equivalent of **1 amu** can be determined by inserting this quantity of mass into Einstein's equation and applying conversion factors.
1 amu = 1.6606 x 10^{-27} kg

3.3. Energy Levels of Atoms

The electrons that circle the nucleus move in fairly well-defined orbits. Some of these electrons are more tightly bound in the atom than others. For example, only 7.38 eV is required to remove the outermost electron from a lead atom, while 88,000 eV is required to remove the innermost electron.

The process of removing an electron from an atom is called **ionization,** and the energy required to remove the electron is called the ionization energy. In a neutral atom (number of electrons **= Z**) it is possible for the electrons to be in a variety of different orbits, each with a different energy level. The state of lowest energy is the one in which the atom is normally found and is called the ground state. When the atom possesses more energy than its ground state energy, it is said to be in an excited state.

An atom cannot stay in the excited state for an indefinite period of time. An excited atom will eventually transition to either a lower-energy excited state, or directly to its ground state, by emitting a discrete bundle of electromagnetic energy called an x-ray. The energy of the x-ray will be equal to the difference between the energy levels of the atom and will typically range from several eV to 100,000 eV in magnitude.

3.4. Energy Levels of the Nucleus

The nucleons in the nucleus of an atom, like the electrons that circle the nucleus, exist in shells that correspond to energy states. The energy shells of the nucleus are less defined and less understood than those of the electrons.

There is a state of lowest energy (the ground state) and discrete possible excited states for a nucleus. Where the discrete energy states for the electrons of an atom are measured in **eV** or **keV,** the energy levels of the nucleus are considerably greater and typically measured in **MeV.**

A nucleus that is in the excited state will not remain at that energy level for an indefinite period. Like the electrons in an excited atom, the nucleons in an excited nucleus will transition towards their lowest energy configuration and in doing so emit a discrete bundle of electromagnetic radiation called a gamma ray (g-ray). The only differences between x-rays and g-rays are their energy levels and whether they are emitted from the electron shell or from the nucleus.

SECTION FOUR: MODES OF RADIOACTIVE DECAY

Most atoms found in nature are stable and do not emit particles or energy that change form over time. Some atoms, however, do not have stable nuclei. These atoms emit radiation in order to achieve a more stable configuration.

4.1 Stability of Nuclei

As mass numbers become larger, the ratio of neutrons to protons in the nucleus becomes larger for the stable nuclei. Non-stable nuclei may have an excess or deficiency of neutrons and undergo a transformation process known as **beta** (β) decay.

Non-stable nuclei can also undergo a variety of other processes such as alpha (α) or neutron (**n**) decay. As a result of these decay processes, the final nucleus is in a more stable or more tightly bound configuration.

4.2 Natural Radioactivity

In 1896, the French physicist Becquerel discovered that crystals of a uranium salt emitted rays that were similar to x-rays in that they were highly penetrating, could affect a photographic plate, and induced electrical conductivity in gases. Becquerel's discovery was followed in 1898 by the identification of two other radioactive elements, **polonium** and **radium,** by Pierre and Marie Curie.

Heavy elements, such as **uranium** or **thorium**, and their unstable decay chain elements emit radiation in their naturally occurring state. **Uranium** and **thorium**, present since their creation at the beginning of geological time, have an extremely slow rate of decay. All naturally occurring nuclides with atomic numbers greater than 82 are radioactive.

4.3 Nuclear Decay

Whenever a nucleus can attain a more stable (i.e., more tightly bound) configuration by emitting radiation, a spontaneous disintegration process known as radioactive decay or nuclear decay may occur. In practice, this

"radiation" may be electromagnetic radiation, particles, or both. Detailed studies of radioactive decay and nuclear reaction processes have led to the formulation of useful conservation principles.

The four principles of most interest in this white paper are discussed below.

1. **Conservation of electric charge:** Conservation of electric charge implies that sum of the charges in the beginning of a process is equal to the sum of the charges after the interaction has occurred.
2. **Conservation of mass number:** Conservation of mass number does not allow a net change in the number of nucleons. However, the conversion of a proton to a neutron and vice versa is allowed.
3. **Conservation of mass and energy:** Implies that the total of the kinetic energy and the energy equivalent of the mass in a system must be conserved in all decays and reactions. Mass can be converted to energy and energy can be converted to mass, but the sum of mass and energy must be constant.
4. **Conservation of momentum:** is responsible for the distribution of the available kinetic energy among product nuclei, particles, and/or radiation. The total amount is the same before and after the reaction even though it may be distributed differently among entirely different nuclides and/or particles.

4.4 Alpha Decay (α)

Alpha decay is the emission of **alpha particles** (**helium nuclei**) which may be represented as either **He^4** or **(α)**. When an unstable nucleus ejects an alpha particle, the atomic number is reduced by 2 and the mass number decreased by 4. An example is uranium-234, (**U^{234}**) which decays by the ejection of an alpha particle accompanied by the emission of a 0.068 MeV **gamma ray photon**.

The combined **kinetic energy** of the new nucleus (Thorium-230, **Th ^{230}**) and the α particle is designated as **KE.** The sum of the **KE** and the **gamma energy** is equal to the difference in mass between the original nucleus **U^{234}** (Uranium-234) and the final particles (equivalent to the binding energy released, since **Δm = BE).** The alpha particle will carry off as much as 98% of the kinetic energy and, in most cases, can be considered to carry off all the kinetic energy.

4.5 Beta Decay (β)

Beta decay is the emission of electrons of nuclear rather than orbital origin. These particles are electrons that have been expelled by excited nuclei and may have a charge of either sign. If both energy and momentum are to be conserved, a third type of particle, the **neutrino** must be involved.

The **neutrino** is associated with positive electron emission, and its antiparticle, the **antineutrino**, , is emitted with a negative electron. These uncharged particles have only the weakest interaction with matter, no mass, and travel at the speed of light. For all practical purposes, they pass through all materials with so few interactions that the energy they possess cannot be recovered.

The **neutrinos** and **antineutrinos** are included here only because they carry a portion of the kinetic energy that would otherwise belong to the beta particle, and therefore, must be considered for energy and momentum to be conserved. They are normally ignored since they are not significant in the context of nuclear reactor applications.

Negative electron emission, effectively converts a neutron to a proton, thus increasing the atomic number by one and leaving the mass number unchanged. This is a common mode of decay for nuclei with an excess of neutrons.

Positively charged electrons (beta-plus) are known as **positrons.** Except for sign, they are nearly identical to their negatively charged cousins. When a positron is ejected from the nucleus, the atomic number is decreased by one and the mass number remains unchanged. A proton has been converted to a neutron.

4.6 Electron Capture (EC, K-capture)

Nuclei having an excess of protons may capture an electron from one of the inner orbits which immediately combines with a proton in the nucleus to form a neutron. This process is called **electron capture** (**EC**). The electron is normally captured from the innermost orbit (the **K-shell**), and, consequently, this process is sometimes called **K-capture**.

A neutrino is formed at the same time that the neutron is formed, and energy carried off by it serves to conserve momentum. Any energy that is available due to the atomic mass of the product being appreciably less than that of the parent will appear as gamma radiation. Also, there will always be characteristic x-rays given off when an electron from one of the higher energy shells moves in to fill the vacancy in the K-shell. Electron capture and positron emission result in the production of the same daughter product, and they exist as competing processes.

For positron emission to occur, however, the mass of the daughter product must be less than the mass of the parent by an amount equal to at least twice the mass of an electron. This mass difference between the parent and daughter is necessary to account for two items present in the parent but not in the daughter. One item is the positron ejected from the nucleus of the parent. The other item is that the daughter product has one less orbital electron than the parent. If this requirement is not met, then orbital electron capture takes place exclusively.

4.7 Gamma Emission (γ)

Gamma radiation is a high-energy electromagnetic radiation that originates in the nucleus. It is emitted in the form of **photons,** discrete bundles of energy that have both wave and particle properties. Often a daughter nuclide is left in an excited state after a radioactive parent nucleus undergoes a transformation by **alpha decay**, **beta decay**, or **electron capture**. The nucleus will drop to the ground state by the emission of gamma radiation.

4.8 Internal Conversion

The usual method for an excited nucleus to go from the excited state to the ground state is by emission of **gamma radiation**. However, in some cases the gamma ray (photon) emerges from the nucleus only to interact with one of the innermost orbital electrons and, as a result, the energy of the photon is transferred to the electron. The gamma ray is then said to have undergone **internal conversion**.

The conversion electron is ejected from the atom with kinetic energy equal to the gamma energy minus the binding energy of the orbital electron. An orbital electron then drops to a lower energy state to fill the vacancy, and this is accompanied by the emission of characteristic x-rays.

4.9 Isomers and Isomeric Transition

Isomeric transition commonly occurs immediately after particle emission; however, the nucleus may remain in an excited state for a measurable period of time before dropping to the ground state at its own characteristic rate. A nucleus that remains in such an excited state is known as a **nuclear isomer** because it differs in energy and behavior from other nuclei with the same atomic number and mass number.

The decay of an excited nuclear isomer to a lower energy level is called an **isomeric transition**. It is also possible for the excited isomer to decay by some alternate means, for example, by beta emission.

4.10 Decay Chains

When an unstable nucleus decays, the resulting daughter nucleus is not necessarily stable. The nucleus resulting from the decay of a parent is often itself unstable, and will undergo an additional decay. This is especially common among the larger nuclides.

It is possible to trace the steps of an unstable atom as it goes through multiple decays trying to achieve stability. The list of the original unstable nuclide, the nuclides that are involved as intermediate steps in the decay, and the final stable nuclide is known as the **decay chain**.

4.11 Predicting Type of Decay

Radioactive nuclides tend to decay in a way that results in a daughter nuclide that lies closer to the line of stability. Due to this, it is possible to predict the type of decay that a nuclide will undergo based on its location relative to the line of stability.

SECTION FIVE: RADIOACTIVITY

The rate at which a sample of radioactive material decays is not constant. As individual atoms of the material decay, there are fewer of those types of atoms remaining. Since the rate of decay is directly proportional to the number of atoms, the rate of decay will decrease as the number of atoms decreases.

5.1 Definition of Radioactivity

Radioactivity is the property of certain nuclides of spontaneously emitting particles or gamma radiation. The decay of radioactive nuclides occurs in a random manner, and the precise time at which a single nucleus will decay cannot be determined. However, the average behavior of a very large sample can be predicted accurately by using statistical methods. These studies have revealed that there is a certain probability that in a given time interval a certain fraction of the nuclei within a sample of a particular nuclide will decay. This probability per unit time that an atom of a nuclide will decay is known as the **radioactive decay constant**. The units for the decay constant are inverse time such as 1/second, 1/minute, 1/hour, or 1/year.

5.2 Activity

The **activity (A)** of a sample is the rate of decay of that sample. This rate of decay is usually measured in the number of disintegrations that occur per second. For a sample containing millions of atoms, the activity is the product of the decay constant and the number of atoms present in the sample.

The relationship between the activity, number of atoms, and decay constant is shown below;

$$\mathbf{A = \lambda N}$$

where:

A = Activity of the nuclide (disintegrations/second)

λ = decay constant of the nuclide (second-1)

N = Number of atoms of the nuclide in the sample

Since λ is a constant, the activity and the number of atoms are always proportional.

5.3 Units of Measurement for Radioactivity

Two common units to measure the activity of a substance are the **curie** (**Ci**) and **becquerel** (**Bq**). A **curie** is a unit of measure of the rate of radioactive decay equal to 3.7×10^{10} disintegrations per second. This is approximately equivalent to the number of disintegrations that one gram of radium-226 will undergo in one second. A **becquerel** is a more fundamental unit of measure of radioactive decay that is equal to 1 disintegration per second.

Currently, the **curie** is more widely used in the United States, but usage of the **becquerel** can be expected to broaden as the metric system slowly comes into wider use. The conversion between curies and becquerels is shown below.

$$1 \text{ curie} = 3.7 \times 10^{10} \text{ becquerels}$$

5.4 Variation of Radioactivity Over Time

The rate at which a given radionuclide sample decays is stated in section **5.2** as being equal to the product of the number of atoms and the decay constant.

From this basic relationship it is possible to use calculus to derive an expression which can be used to calculate how the number of atoms present will change over time. The derivation is beyond the scope of this white paper but the following equation is the useful result of the solution of this important differential equation:

$$N_t = N_o e^{-\lambda t}$$

where:

N_t = number of atoms present at time t
N_o = number of atoms initially present o
λ= decay constant (time-1)
t = time

5.5 Radioactive Half-Life

One of the most useful terms for estimating how quickly a nuclide will decay is the **radioactive half-life**. The **radioactive half-life** is defined as the amount of time required for the activity to decrease to one-half of its original value.

A relationship between the half-life and decay constant can be developed from the equation developed in section **5.4.**
Assuming an initial number of atoms N_o the population, and consequently, the activity may be noted to decrease by one-half of this value in a time of one half-life. Additional decreases occur so that whenever one half-life elapses, the number of atoms drops to one-half of what its value was at the beginning of that time interval. After five half-lives have elapsed, only 1/32, or 3.1%, of the original number of atoms remains. After seven half-lives, only 1/128, or 0.78%, of the atoms remains. The number of atoms existing after 5 to 7 half-lives can usually be assumed to be negligible.

5.6 Plotting Radioactive Decay

It is useful to plot the activity of a nuclide as it changes over time. Plots of this type can be used to determine when the activity will fall below a certain level. This plot is usually done showing activity on either a linear or a logarithmic scale. The decay of the activity of a single nuclide on a logarithmic scale will plot as a straight line because the decay is exponential. If a substance contains more than one radioactive nuclide, the total activity is the sum of the individual activities of each nuclide. The initial activity of each of the nuclides would be the product of the number of atoms and the decay constant.

5.7 Radioactive Equilibrium

A. **Radioactive equilibrium** exists when a radioactive nuclide is decaying at the same rate at which it is being produced. Since the production rate and

decay rate are equal, the number of atoms present remains constant over time.

B. Transient radioactive equilibrium occurs when the parent nuclide and the daughter nuclide decay at essentially the same rate. For transient equilibrium to occur, the parent must have a long half-life when compared to the daughter. An example of this type of compound decay process is barium-140, which decays by beta emission to **lanthanum-140,** which in turn decays by beta emission to stable **cerium-140**.

The decay constant for **barium-140** is considerably smaller than the decay constant for **lanthanum-140**. Remember that the rate of decay of both the parent and daughter can be represented as **λN**. Although the decay constant for **barium-140** is smaller, the actual rate of decay (**λN)** is initially larger than that of **lanthanum-140** because of the great difference in their initial concentrations. As the concentration of the daughter increases, the rate of decay of the daughter will approach and eventually match the decay rate of the parent. When this occurs, they are said to be in transient equilibrium.

C. Secular equilibrium occurs when the parent has an extremely long half-life. In the long decay chain for a naturally radioactive element, such as thorium-232, where all of the elements in the chain are in secular equilibrium, each of the descendants has built up to an equilibrium amount and all decay at the rate set by the original parent. The only exception is the final stable element on the end of the chain. Its number of atoms is constantly increasing.

SECTION SIX: NEUTRON INTERACTIONS

Neutrons can cause many different types of interactions. The neutron may simply scatter off the nucleus in two different ways, or it may actually be absorbed into the nucleus. If a neutron is absorbed into the nucleus, it may result in the emission of a gamma ray or a subatomic particle, or it may cause the nucleus to fission.

6.1 Scattering

A neutron **scattering** reaction occurs when a nucleus, after having been struck by a neutron, emits a single neutron. Despite the fact that the initial and final neutrons do not need to be (and often are not) the same, the net effect of the reaction is as if the projectile neutron had merely "bounced off," or scattered from, the nucleus. The two categories of scattering reactions, **elastic** and **inelastic** scattering, are described below:

A. Elastic Scattering

In an **elastic scattering** reaction between a neutron and a target nucleus, there is no energy transferred into nuclear excitation. Momentum and

kinetic energy of the "system" are conserved although there is usually some transfer of kinetic energy from the neutron to the target nucleus. The target nucleus gains the amount of kinetic energy that the neutron loses. Elastic scattering of neutrons by nuclei can occur in two ways:

1. The more unusual of the two interactions is the absorption of the neutron, forming a compound nucleus, followed by the re-emission of a neutron in such a way that the total kinetic energy is conserved and the nucleus returns to its ground state. This is known as **resonance elastic scattering** and is very dependent upon the initial kinetic energy possessed by the neutron. Due to formation of the compound nucleus, it is also referred to as compound elastic scattering.

2. The second, more usual method, is termed **potential elastic scattering** and can be understood by visualizing the neutrons and nuclei to be much like billiard balls with impenetrable surfaces. Potential scattering takes place with incident neutrons that have an energy of up to about **1 MeV.** In potential scattering, the neutron does not actually touch the nucleus and a compound nucleus is not formed. Instead, the neutron is acted on and scattered by the short range nuclear forces when it approaches close enough to the nucleus.

B. Inelastic Scattering

In **inelastic scattering,** the incident neutron is absorbed by the target nucleus, forming a compound nucleus. The compound nucleus will then emit a neutron of lower kinetic energy which leaves the original nucleus in an excited state. The nucleus will usually, by one or more gamma emissions, emit this excess energy to reach its ground state.

For the nucleus that has reached its ground state, the sum of the kinetic energy of the exit Inelastic Scattering neutron, the target nucleus, and the total gamma energy emitted is equal to the initial kinetic energy of the incident neutron.

6.2 Absorption Reactions

Most **absorption reactions** result in the loss of a neutron coupled with the production of a charged particle or gamma ray. When the product nucleus is radioactive, additional radiation is emitted at some later time. **Radiative capture, particle ejection**, and **fission** are all categorized as absorption reactions and are briefly described below.

A. Radiative Capture: In radiative capture the incident neutron enters the target nucleus forming a compound nucleus. The compound nucleus then decays to its ground state by gamma emission.

B. Particle Ejection: In a **particle ejection** reaction the incident particle enters the target nucleus forming a compound nucleus. The newly formed compound nucleus has been excited to a high enough energy level to

cause it to eject a new particle while the incident neutron remains in the nucleus. After the new particle is ejected, the remaining nucleus may or may not exist in an excited state depending upon the mass-energy balance of the reaction.

C. Fission: One of the most important interactions that neutrons can cause is fission, in which the nucleus that absorbs the neutron actually splits into two similarly sized parts. Fission will be discussed in detail in the next chapter.

SECTION SEVEN: NUCLEAR FISSION

Nuclear fission is a process in which an atom splits and releases energy, fission products, and neutrons. The neutrons released by fission can, in turn, cause the fission of other atoms.

7.1 Fission

In the fission reaction the incident neutron enters the heavy target nucleus, forming a compound nucleus that is excited to such a high energy level (E > E) that the nucleus "splits" (fissions) into two large fragments plus some neutrons. A large amount of energy is released in the form of radiation and fragment kinetic energy.

7.2. Liquid Drop Model of a Nucleus

The nucleus is held together by the attractive nuclear force between nucleons. The characteristics of the nuclear force are listed below:

1. Very short range, with essentially no effect beyond nuclear dimensions (10^{-13} cm)
2. Stronger than the repulsive electrostatic forces within the nucleus
3. Independent of nucleon pairing, in that the attractive forces between pairs of neutrons are no different than those between pairs of protons or a neutron and a proton
4. Saturable, that is, a nucleon can attract only a few of its nearest neighbors

One theory of fission considers the fissioning of a nucleus similar in some respects to the splitting of a liquid drop. This analogy is justifiable to some extent by the fact that a liquid drop is held together by molecular forces that tend to make the drop spherical in shape and that try to resist any deformation in the same manner as nuclear forces are assumed to hold the nucleus together.

By considering the nucleus as a liquid drop, the fission process can be described. the nucleus in the ground state is undistorted, and its attractive nuclear forces are greater than the repulsive electrostatic forces between the protons within the nucleus. When an incident particle (in this instance a neutron) is absorbed by the target nucleus, a compound nucleus is formed.

The compound nucleus temporarily contains all the charge and mass involved in the reaction and exists in an excited state. The excitation energy added to the compound nucleus is equal to the binding energy contributed by the incident particle plus the kinetic energy possessed by that particle.
The excitation energy thus imparted to the compound nucleus, which may cause it to oscillate and become distorted. If the excitation energy is greater than a certain critical energy, the oscillations may cause the compound nucleus to become dumbbell-shaped. When this happens, the attractive nuclear forces (short-range) in the neck area are small due to saturation, while the repulsive electrostatic forces (long-range) are only slightly less than before. When the repulsive electrostatic forces exceed the attractive nuclear forces, nuclear fission occurs,

7.3. Critical Energy

The measure of how far the energy level of a nucleus is above its ground state is called the **excitation energy**. For fission to occur, the excitation energy must be above a particular value for that nuclide. The **critical energy** (Ecrit) is the minimum excitation energy required for fission to occur.

7.4. Fissionable, Fissile and Fertile Materials

Theoretically, all nuclei heavier than iron have the potential to undergo fission, however the energy barrier that needs to be exceeded before fission can occur is impossibly high for all but the heavier elements. It is only for mass numbers greater than about 230 that the fission activation energy may be less than 10 MeV.

A. Fissionable Materials

Consider the compound nucleus Uranium 239 formed by the absorption of a neutron by $\mathbf{U^{238}}$**.** For the neutron to induce fission the sum of the binding and kinetic energy transferred to the $\mathbf{U^{239}}$ compound nucleus must exceed its fission activation energy. The activation energy of $\mathbf{U^{239}}$ is 7 MeV; the difference between the binding energy of $\mathbf{U^{238}}$ and $\mathbf{U^{239}}$ is 5.5 MeV. Thus the kinetic energy of the incoming neutron must be at least 1.5 MeV.

Materials, such as $\mathbf{U^{238}}$**,** which may undergo fission following absorption of fast neutrons of a few MeV kinetic energy are called **Fissionable Materials.**

B. Fissile Materials

A **fissile material** is composed of nuclides for which fission is possible with neutrons of any energy level. What is especially significant about these nuclides is their ability to be fissioned with zero kinetic energy neutrons (thermal neutrons). Thermal neutrons have very low kinetic energy levels (essentially zero) because they are roughly in equilibrium with the thermal motion of surrounding materials.

Therefore, in order to be classified as fissile, a material must be capable of fissioning after absorbing a thermal neutron. Consequently, they impart essentially no kinetic energy to the reaction. Fission is possible in these materials with thermal neutrons, since the change in binding energy supplied by the neutron addition alone is high enough to exceed the critical energy. Some examples of fissile nuclides are **U^{235}** (uranium-235), **U^{233}** (uranium-233), and **PU^{239}** (plutonium-239).

Consider the compound nucleus **U^{236}** formed from the absorption of a neutron by **U^{235}.** In this case the **U^{236}** fission activation energy is 6.5 MeV whereas the difference in binding energy between **U^{235}** and **U^{236}** is 6.8 MeV. Thus neutrons of any kinetic energy can induce fission following absorption in **U^{235}**. **U^{235}** is the only naturally occurring fissile material.

C. Fertile Materials

All of the neutron absorption reactions that do not result in fission lead to the production of new nuclides through the process known as **transmutation.** These nuclides can, in turn, be transmuted again or may undergo radioactive decay to produce still different nuclides. The nuclides that are produced by this process are referred to as transmutation products. Because several of the fissile nuclides do not exist in nature, they can only be produced by nuclear reactions (transmutation).

The target nuclei for such reactions are said to be fertile. **Fertile materials** are materials that can undergo transmutation to become fissile materials. The fertile nuclides, thorium-232 and uranium-238 can be bombarded with neutrons to produce uranium-233 and plutonium-239, respectively.

If a reactor contains fertile material in addition to its fissile fuel, some new fuel will be produced as the original fuel is burned up. This is called **conversion**. Reactors that are specifically designed to produce fissionable fuel are called "breeder" reactors. In such reactors, the amount of fissionable fuel produced is greater than the amount of fuel burnup. If less fuel is produced than used, the process is called conversion, and the reactor is termed a "converter."

A **fissionable material** is composed of nuclides for which fission with neutrons is possible. All fissile nuclides fall into this category. However, also included are those nuclides that can be fissioned only with high energy neutrons. The change in binding energy that occurs as the result of neutron absorption results in a nuclear excitation energy level that is less than the required critical energy. Therefore, the additional excitation energy must be supplied by the kinetic energy of the incident neutron.

The reason for this difference between fissile and fissionable materials is the so-called odd-even effect for nuclei. It has been observed that nuclei

with even numbers of neutrons and/or protons are more stable than those with odd numbers. Therefore, adding a neutron to change a nucleus with an odd number of neutrons to a nucleus with an even number of neutrons produces an appreciably higher binding energy than adding a neutron to a nucleus already possessing an even number of neutrons. Some examples of nuclides requiring high energy neutrons to cause fission are **Th^{232}** (**thorium-232)**, **U^{238}** (**uranium-238)**, and **Pu^{240}** (**plutonium-240)**.

Uranium 239, which may be formed as the result of **U^{238}** absorbing a neutron, is radioactive and decays by β emission, with a half-life of 23-1/2 minutes, to **Neptunium 239.** This neptunium 239 decays by β emission, with half life of 2.3 days, to **Plutonium 239**, an α emitter of half life of 24,000 years. It turns out that plutonium 239 is a fissile material; that is, as in the case of **U^{235}**, it readily undergoes fission on absorption of neutrons of any energy including slow neutrons of very low energies. The uranium 238 is called a **Fertile Material** because the absorption of the neutrons, which we have seen previously it most readily does in the resonance capture mode, leads to the formation of the fissile material **Pu^{239}.**

Similarly thorium 232 is also a fertile material because neutron absorption leads, via **Protactinium 233,** to the fissile material **Uranium 233.**

Thus the fertile materials **U^{238}** and **Th^{232}** yield the fissile materials **Pu^{239}** and **U^{233}**, respectively.

7.5 Natural Uranium

Natural Uranium is found in ore deposits in many places around the world. It is predominantly a mixture of the two isotopes 238, 234 and 235, in the proportions mentioned at the beginning of this white paper. All three isotopes are radioactive.
Therefore of the three fissile materials mentioned above, natural uranium is a direct source for one, **U^{235},** and an indirect source for a second, **Pu^{239}** via the fertile **U^{238}.** These facts underscore the importance of natural uranium in the production of Nuclear Power.

The third fissile material, **U^{233},** is of little significance at present, although of possibly important potential because of large ore reserves of the fertile thorium.

Before returning to our discussion on fission it will be useful to summarize some of the properties of natural uranium and its isotopes:

- Natural uranium consists of
  99.3% **U^{238}** α emitter half life 4.5×10^9 years
  0.7% **U^{235}** α emitter half life 7.1×10^8 years
  0.1% **U^{234}** α emitter half life 2.5×10^5 years

- **U^{238}** is a fissionable material; it can undergo fission provided the absorbed neutron has an incident kinetic energy of at least 1.1 MeV.

- **U^{238}** is a fertile material, forming fissile **Pu^{239}** following capture of a neutron. Neutrons of intermediate energy are readily captured in the resonance capture peaks of **U^{238}.**

- **U^{235}** is a fissile material; it can undergo fission with neutrons of any energy but is much more likely to do so the less energetic, or slower, the neutron.

The isotope uranium-235 is usually the desired material for use in reactors. A vast amount of equipment and energy are expended in processes that separate the isotopes of uranium (and other elements). The details of these processes are beyond the scope of this white paper. These processes are called enrichment processes because they selectively increase the proportion of a particular isotope. The enrichment process typically starts with feed material that has the proportion of isotopes that occur naturally.

In the case of uranium, the natural uranium ore is 0.72 a/o uranium-235. The desired outcome of the enrichment process is to produce enriched uranium.

- **Enriched uranium** is defined as uranium in which the isotope uranium-235 has a concentration greater than its natural value. The enrichment process will also result in the byproduct of depleted uranium.

- **Depleted uranium** is defined as uranium in which the isotope uranium-235 has a concentration less than its natural value. Although depleted uranium is referred to as a by-product of the enrichment process, it does have uses in the nuclear field and in commercial and defense industries.

7.6 Critical Energies Compared to Binding Energy of Last Neutron

Uranium-235 fissions with thermal neutrons because the binding energy released by the absorption of a neutron is greater than the critical energy for fission; therefore uranium-235 is a fissile material. The binding energy released by uranium-238 absorbing a thermal neutron is less than the critical energy, so additional energy must be possessed by the neutron for fission to be possible. Consequently, uranium-238 is a fissionable material.

7.7 Binding Energy Per Nucleon (BE/A)

As the number of particles in a nucleus increases, the total binding energy also increases. The rate of increase, however, is not uniform. This lack of uniformity results in a variation in the amount of binding energy associated with each nucleon within the nucleus. This variation in the binding energy per

nucleon (BE/A) is easily seen when the average BE/A is plotted versus atomic mass number (A).
This plot illustrates that as the atomic mass number increases, the binding energy per nucleon decreases for A > 60. The BE/A curve reaches a maximum value of 8.79 MeV at A = 56 and decreases to about 7.6 MeV for A = 238. The general shape of the BE/A curve can be explained using the general properties of nuclear forces. The nucleus is held together by very short-range attractive forces that exist between nucleons. On the other hand, the nucleus is being forced apart by long range repulsive electrostatic (coulomb) forces that exist between all the protons in the nucleus.

As the atomic number and the atomic mass number increase, the repulsive electrostatic forces within the nucleus increase due to the greater number of protons in the heavy elements. To overcome this increased repulsion, the proportion of neutrons in the nucleus must increase to maintain stability. This increase in the neutron-to-proton ratio only partially compensates for the growing proton-proton repulsive force in the heavier, naturally occurring elements. Because the repulsive forces are increasing, less energy must be supplied, on the average, to remove a nucleon from the nucleus. The BE/A has decreased. The BE/A of a nucleus is an indication of its degree of stability. Generally, the more stable nuclides have higher BE/A than the less stable ones. The increase in the BE/A as the atomic mass number decreases from 260 to 60 is the primary reason for the energy liberation in the fission process.

The heaviest nuclei require only a small distortion from a spherical shape (small energy addition) for the relatively large coulomb forces forcing the two halves of the nucleus apart to overcome the attractive nuclear forces holding the two halves together. Consequently, the heaviest nuclei are easily fissionable compared to lighter nuclei.

SECTION EIGHT: ENERGY RELEASE FROM FISSION

Fission of heavy nuclides converts a small amount of mass into an enormous amount of energy. The amount of energy released by fission can be determined based on either the change in mass that occurs during the reaction or by the difference in binding energy per nucleon between the fissile nuclide and the fission products.

8.1. Calculation of Fission Energy

Nuclear fission results in the release of enormous quantities of energy. It is necessary to be able to calculate the amount of energy that will be produced. The logical manner in which to pursue this is to first investigate a typical fission reaction. When the compound nucleus splits, it breaks into two fission fragments, rubidium-93, cesium-140, and some neutrons. Both fission products then decay by multiple - emissions as a result of the high neutron-to-proton ratio possessed by these nuclides.

In most cases, the resultant fission fragments have masses that vary widely. The most probable pair of fission fragments for the thermal fission of the fuel uranium-235 have masses of about 95 and 140.

Referring now to the binding energy per nucleon curve, we can estimate the amount of energy released by our "typical" fission by plotting this reaction on the curve and calculating the change in binding energy (DBE) between the reactants on the left-hand side of the fission equation and the products on the right-hand side. Plotting the reactant and product nuclides on the curve shows that the total binding energy of the system after fission is greater than the total binding energy of the system before fission. When there is an increase in the total binding energy of a system, the system has become more stable by releasing an amount of energy equal to the increase in total binding energy of the system. Therefore, in the fission process, the energy liberated is equal to the increase in the total binding energy of the system.

8.2 Binding Energy per Nucleon

The energy released will be equivalent to the difference in binding energy (**BE**) between the reactants and the products. The energy liberation during the fission process can also be explained from the standpoint of the conservation of mass-energy. During the fission process, there is a decrease in the mass of the system. There must, therefore, be energy liberated equal to the energy equivalent of the mass lost in the process.

Again, referring to the "typical" fission reaction. E , the instantaneous energy, is the energy released immediately after the fission process. It is equal to the energy equivalent of the mass lost in the fission process. The total energy released per fission will vary from the fission to the next depending on what fission products are formed, but the average total energy released per fission of uranium-235 with a thermal neutron is **200 MeV.**

The majority of the energy liberated in the fission process is released immediately after the fission occurs and appears as the kinetic energy of the fission fragments, kinetic energy of the fission neutrons, and instantaneous gamma rays. The remaining energy is released over a period of time after the fission occurs and appears as kinetic energy of the beta, neutrino, and decay gamma rays.

8.3. Estimation of Decay Energy

In addition to this instantaneous energy release during the actual fission reaction, there is additional energy released when the fission fragments decay by - emission. This additional energy is called decay energy, E . The energy released during the decay for each chain will be equivalent to the mass difference between the original fission product and the sum of the final stable nuclide and the beta particles emitted.

8.4. Distribution of Fission Energy

The average energy distribution for the energy released per fission with a thermal neutron in uranium-235 is shown below:

A. Instantaneous Energy from Fission

Kinetic Energy of Fission Products 167 Mev
Energy of Fission Neutrons 5 MeV
Instantaneous Gamma-ray Energy 5 MeV
Capture Gamma-ray Energy 10 MeV
Total Instantaneous Energy 187 MeV

B. Delayed Energy from Fission

Beta Particles From Fission Products 7 Mev
Gamma-rays from Fission Products 6 MeV
Neutrinos 10 MeV
Total Delayed Energy 23 MeV

All of the energy released, with the exception of the neutrino energy, is ultimately transformed into heat through a number of processes. The fission fragments, with their high positive charge and kinetic energy, cause ionization directly as they rip orbital electrons from the surrounding atoms. In this ionization process, kinetic energy is transferred to the surrounding atoms of the fuel material, resulting in an increase in temperature. The beta particles and gamma rays also give up their energy through ionization, and the fission neutrons interact and lose their energy through elastic scattering.

Of the 200 MeV released per fission, about seven percent (13 MeV) is released at some time after the instant of fission. When a reactor is shut down, fissions essentially cease, but energy is still being released from the decay of fission products. The heat produced by this decay energy is referred to as "decay heat." Although decay energy represents about seven percent of reactor heat production during reactor operation, once the reactor is shut down the decay heat production drops off quickly to a small fraction of its value while operating.

SECTION NINE: INTERACTION OF RADIATION WITH MATTER

Different types of radiation interact with matter in widely different ways. A large, massive, charged alpha particle cannot penetrate a piece of paper and even has a limited range in dry air. A neutrino, at the other extreme, has a low probability of interacting with any matter, even if it passed through the diameter of the earth.

9.1 Ionization

Radiation can be classified into two general groups, charged and uncharged; therefore, it may be expected that interactions with matter fall into two general types. Charged particles directly ionize the media through which they pass, while uncharged particles and photons can cause ionization only indirectly or by secondary radiation.

A moving charged particle has an electrical field surrounding it, which interacts with the atomic structure of the medium through which it is passing. This interaction decelerates the particle and accelerates electrons in the atoms of the medium. The accelerated electrons may acquire enough energy

to escape from the parent atom. This process, whereby radiation "strips" off orbital electrons, is called ionization. Uncharged moving particles have no electrical field, so they can only lose energy and cause ionization by such means as collisions or scattering. A photon can lose energy by the photoelectric effect, Compton effect, or pair production.

Because ionizing radiation creates ions in pairs, the intensity of ionization or the specific ionization is defined as the number of ion-pairs formed per centimeter of travel in a given material. The amount of ionization produced by a charged particle per unit path length, which is a measure of its ionizing power, is roughly proportional to the particle's mass and the square of its charge as illustrated in the equation below. where:

$$\mathbf{I} = \frac{\mathbf{mz^2}}{\mathbf{K.E.}}$$

Where:
I is the ionizing power
m is the mass of the particle
z is the number of unit charges it carries
K.E. is its kinetic energy

Since m for an alpha particle is about 7300 times as large as m for a beta article, and z is twice as great, an alpha will produce much more ionization per unit path length than a beta particle of the same energy. This phenomenon occurs because the larger alpha particle moves slower for a given energy and thus acts on a given electron for a longer time.

9.2 Alpha Radiation

Alpha radiation is normally produced from the radioactive decay of heavy nuclides and from certain nuclear reactions. The alpha particle consists of 2 neutrons and 2 protons, so it is essentially the same as the nucleus of a helium atom. Because it has no electrons, the alpha particle has a charge of +2. This positive charge causes the alpha particle to strip electrons from the orbits of atoms in its vicinity. As the alpha particle passes through material, it removes electrons from the orbits of atoms it passes near. Energy is required to remove electrons and the energy of the alpha particle is reduced by each reaction. Eventually the particle will expend its kinetic energy, gain 2 electrons in orbit, and become a helium atom. Because of its strong positive charge and large mass, the alpha particle deposits a large amount of energy in a short distance of travel. This rapid, large deposition of energy limits the penetration of alpha particles. The most energetic alpha particles are stopped by a few centimeters of air or a sheet of paper.

9.3 Beta-Minus Radiation

A beta-minus particle is an electron that has been ejected at a high velocity from an unstable nucleus. An electron has a small mass and an electrical

charge of -1. Beta particles cause ionization by displacing electrons from atom orbits. The ionization occurs from collisions with orbiting electrons. Each collision removes kinetic energy from the beta particle, causing it to slow down. Eventually the beta particle will be slowed enough to allow it to be captured as an orbiting electron in an atom. Although more penetrating than the alpha, the beta is relatively easy to stop and has a low power of penetration. Even the most energetic beta radiation can be stopped by a few millimeters of metal.

9.4 Positron Radiation

Positively charged electrons are called positrons. Except for the positive charge, they are identical to beta-minus particles and interact with matter in a similar manner. Positrons are very short-lived, however, and quickly are annihilated by interaction with a negatively charged electron, producing two gammas with a combined energy (calculated below) equal to the rest mass of the positive and negative electrons.

9.5 Neutron Radiation

Neutrons have no electrical charge. They have nearly the same mass as a proton (a hydrogen atom nucleus). A neutron has hundreds of times more mass than an electron, but 1/4 the mass of an alpha particle. The source of neutrons is primarily nuclear reactions, such as fission, but they may also be produced from the decay of radioactive nuclides. Because of its lack of charge, the neutron is difficult to stop and has a high penetrating power.

Neutrons are attenuated (reduced in energy and numbers) by three major interactions, elastic scatter, inelastic scatter, and absorption. In elastic scatter, a neutron collides with a nucleus and bounces off. This reaction transmits some of the kinetic energy of the neutron to the nucleus of the atom, resulting in the neutron being slowed, and the atom receives some kinetic energy (motion). This process is sometimes referred to as "the billiard ball effect."

As the mass of the nucleus approaches the mass of the neutron, this reaction becomes more effective in slowing the neutron. Hydrogenous material attenuates neutrons most effectively. In the inelastic scatter reaction, the same neutron/nucleus collision occurs as in elastic scatter. However, in this reaction, the nucleus receives some internal energy as well as kinetic energy. This slows the neutron, but leaves the nucleus in an excited state.

When the nucleus decays to its original energy level, it normally emits a gamma ray. In the absorption reaction, the neutron is actually absorbed into the nucleus of an atom. The neutron is captured, but the atom is left in an excited state. If the nucleus emits one or more gamma rays to reach a stable level, the process is called radiative capture. This reaction occurs at most neutron energy levels, but is more probable at lower energy levels.

9.6 Gamma Radiation

Gamma radiation is electromagnetic radiation. It is commonly referred to as a gamma ray and is very similar to an x-ray. The difference is that gamma rays are emitted from the nucleus of an atom, and x-rays are produced by orbiting electrons. The x-ray is produced when orbiting electrons move to a lower energy orbit or when fast-moving electrons approaching an atom are deflected and decelerated as they react with the atom's electrical field (called **Bremsstrahlung**). The gamma ray is produced by the decay of excited nuclei and by nuclear reactions. Because the gamma ray has no mass and no charge, it is difficult to stop and has a very high penetrating power. A small fraction of the original gamma stream will pass through several feet of concrete or several meters of water.

There are three methods of attenuating gamma rays:

1. **Photo-Electric Effect:** The first method is referred to as the **photo-electric effect.** When a low energy gamma strikes an atom, the total energy of the gamma is expended in ejecting an electron from orbit. The result is ionization of the atom and expulsion of a high energy electron. This reaction is most predominant with low energy gammas interacting in materials with high atomic weight and rarely occurs with gammas having an energy above 1 MeV. Annihilation of the gamma results. Any gamma energy in excess of the binding energy of the electron is carried off by the electron in the form of kinetic energy.

2. **Compton Scattering:** The second method of attenuation of gammas is called **Compton scattering**. The gamma interacts with an orbital or free electron; however, in this case, the photon loses only a fraction of its energy. The actual energy loss depending on the scattering angle of the gamma. The gamma continues on at lower energy, and the energy difference is absorbed by the electron. This reaction becomes important for gamma energies of about 0.1 MeV and higher.

3. **Pair Production:** At higher energy levels, a third method of attenuation is predominant. This method is **pair-production**. When a high energy gamma passes close enough to a heavy nucleus, the gamma completely disappears, and an electron and a positron are formed. For this reaction to take place, the original gamma must have at least 1.02 MeV energy. Any energy greater than 1.02 MeV becomes kinetic energy shared between the electron and positron. The probability of pair production increases significantly for higher energy

CHAPTER TWO:INTRODUCTION TO NEUTRON PHYSICS

SECTION ONE: NEUTRON SOURCES

Neutrons from a variety of sources are always present in a reactor core. This is true even when the reactor is shut down. Some of these neutrons are produced by naturally occurring (intrinsic) neutron sources, while others may be the result of fabricated (installed) neutron sources that are incorporated into the design of the reactor. The neutrons produced by sources other than neutron-induced fission are often grouped together and classified as source neutrons.

In addition to neutron-induced fission, neutrons are produced by other reactions. The neutrons produced by reactions other than neutron-induced fission are called **source neutrons**. Source neutrons are important because they ensure that the neutron population remains high enough to allow a visible indication of neutron level on the most sensitive monitoring instruments while the reactor is shutdown and during the startup sequence. This verifies instrument operability and allows monitoring of neutron population changes. Source neutrons can be classified as either intrinsic or installed neutron sources.

1.1 Intrinsic Neutron Sources

Some neutrons will be produced in the materials present in the reactor due to a variety of unavoidable reactions that occur because of the nature of these materials. **Intrinsic neutron sources** are those neutron-producing reactions that always occur in reactor materials.

A limited number of neutrons will always be present, even in a reactor core that has never been operated, due to spontaneous fission of some heavy nuclides that are present in the fuel. Uranium-238, uranium-235, and plutonium-239 undergo spontaneous fission to a limited extent. Uranium-238, for example, yields almost 60 neutrons per hour per gram.

1.2 Neutron Production by Spontaneous Fission

Another intrinsic neutron source is a reaction involving natural boron and fuel. In some reactors, natural boron is loaded into the reactor core as a neutron absorber to improve reactor control or increase core life-time. Boron-11 (80.1% of natural boron) undergoes a reaction with the alpha particle emitted by the radioactive decay of heavy nuclides in the fuel to yield a neutron.

The boron-11 must be mixed with, or in very close proximity to, the fuel for this reaction because of the short path length of the alpha particle. For a reactor core with this configuration, this **(α,n**) reaction is an important source of neutrons for reactor startup. In a reactor that has been operated, another source of neutrons becomes significant. Neutrons may be produced by the interaction of a gamma ray and a deuterium nucleus. This reaction is commonly referred to as a photo-neutron reaction because it is initiated by electromagnetic radiation and results in the production of a neutron.

There is an abundant supply of high energy gammas in a reactor that has been operated because many of the fission products are gamma emitters. All water-cooled reactors have some deuterium present in the coolant in the reactor core because a small fraction of natural hydrogen is the isotope deuterium.

The atom percentage of deuterium in the water ranges from close to the naturally occurring value (0.015%) for light water reactors to above 90% deuterium for heavy water reactors. Therefore, the required conditions for production of photo-neutrons exist.

The supply of gamma rays decreases with time after shutdown as the gamma emitters decay; therefore, the photo-neutron production rate also decreases. In a few particular reactors, additional Deuterium Oxide, **DO_2** (heavy water) may be added to the reactor to increase the production of photo-neutrons following a long shutdown period.

1.3 Installed Neutron Sources

Because intrinsic neutron sources can be relatively weak or dependent upon the recent power history of the reactor, many reactors have artificial sources of neutrons installed. These neutron sources ensure that shutdown neutron levels are high enough to be detected by the nuclear instruments at all times. This provides a true picture of reactor conditions and any change in these conditions.

An **installed neutron source** is an assembly placed in or near the reactor for the sole purpose of producing source neutrons. One strong source of neutrons is the artificial nuclide californium-252, which emits neutrons at the rate of about **2×10^{12}** neutrons per second per gram as the result of spontaneous fission. Important drawbacks for some applications may be its high cost and its short half-life (2.65 years).

Many installed neutron sources use the (α**,n**,) reaction with beryllium. These sources are composed of a mixture of metallic beryllium (100% beryllium-9) with a small quantity of an alpha particle emitter, such as a compound of radium, polonium, or plutonium. The reaction that occurs produces a neutron flux. The beryllium is intimately (homogeneously) mixed with the alpha emitter and is usually enclosed in a stainless steel capsule.

Another type of installed neutron source that is widely used is a photo-neutron source that employs the **gamma/neutron**(γ**, n**) reaction with beryllium. Beryllium is used for photo-neutron sources because its stable isotope beryllium-9 has a weakly attached last neutron with a binding energy of only 1.66 MeV. Thus, a gamma ray with greater energy than 1.66 MeV can cause neutrons to be ejected by the (γ**, n**) reaction.

SECTION TWO: NUCLEAR CROSS SECTIONS AND NEUTRON FLUX

To determine the frequency of neutron interactions, it is necessary to describe the availability of neutrons to cause interaction and the probability of a neutron interacting with material. The availability of neutrons and the probability of interaction are quantified by the neutron flux and nuclear cross section.

2.1. Introduction

Fission neutrons are born with an average energy of about 2 MeV. These fast neutrons interact with the reactor core materials in various absorption and scattering reactions. Collisions that result in scattering are useful in slowing neutrons to thermal energies. Thermal neutrons may be absorbed by fissile nuclei to produce more fissions or be absorbed in fertile material for conversion to fissionable fuel. Absorption of neutrons in structural components, coolant, and other non-fuel material results in the removal of neutrons without fulfilling any useful purpose. To safely and efficiently operate a nuclear reactor it is necessary to predict the probability that a particular absorption or scattering reaction will occur. Once these probabilities are known, if the availability of neutrons can be determined, then the rate at which these nuclear reactions take place can be predicted.

2.2. Atom Density

One important property of a material is the atom density. **The atom density** is the number of atoms of a given type per unit volume of the material. To calculate the atom density of a substance use the following equation:

$$\mathbf{N} = \frac{\rho \mathbf{N_A}}{\mathbf{M}}$$

where:

N = atom density (atoms/cm^3)
ρ = density (gram/cm^3)
$\mathbf{N_A}$ = Avogadro's number (6.022 x 10^{23} atoms/mole)
M = gram atomic weight

2.3 Cross-Section for Nuclear Reactions

The probability of a neutron interacting with a nucleus for a particular reaction is dependent upon not only the kind of nucleus involved, but also the energy of the neutron. Accordingly, the absorption of a thermal neutron in most materials is much more probable than the absorption of a fast neutron. Also, the probability of interaction will vary depending upon the type of reaction involved.

The probability of a particular reaction occurring between a neutron and a nucleus is called the **microscopic cross section (σ)** of the nucleus for the particular reaction. This cross section will vary with the energy of the neutron.

The microscopic cross section may also be regarded as the effective area the nucleus presents to the neutron for the particular reaction. The larger the effective area, the greater the probability for reaction. Because the microscopic cross section is an area, it is expressed in units of area, or square centimeters.

A square centimeter is tremendously large in comparison to the effective area of a nucleus, and it has been suggested that a physicist once referred to the measure of a square centimeter as being "as big as a barn" when applied to nuclear processes. The name has persisted and microscopic cross sections are expressed in terms of **barns**. The relationship between **barns** and $\mathbf{cm^2}$ is shown below.

$$\mathbf{1\ barn} = \mathbf{10^{-24}\ cm^2}$$

Whether a neutron will interact with a certain volume of material depends not only on the microscopic cross section of the individual nuclei but also on the number of nuclei within that volume. Therefore, it is necessary to define another kind of cross section known as the **macroscopic cross section (Σ)**.

The **macroscopic cross section** is the probability of a given reaction occurring per unit travel of the neutron.

A. Microscopic Cross Section

For neutrons of a specific energy one or more of the foregoing nuclear reactions can occur. It is necessary to have a method of calculating how many neutrons are undergoing which reactions.

Clearly the rate **R** at which neutrons undergo any reactions is dependent on the number of target nuclei and the number of bombarding neutrons. We are going to say that the rate **R** is proportional to $\mathbf{N\phi}$ where **N** is the number of nuclei per cubic meters for the target material and ϕ is the intensity of the neutron radiation measured in neutrons per square meters. ϕ is called the neutron flux and is the product of the number of neutrons per cubic meters and the average neutron speed, denoted ν, which may be in any direction. We can say that: $\phi = n\nu$ neutrons per square meters. If we let this constant of proportionality by σ, so that:

$$\mathbf{R = \sigma N\phi}$$

This equation is usually written in the order

$$\mathbf{R = N\sigma\phi}$$

And the value of the proportionality constant σ for each type of nuclear reaction is dependent on the target material and the neutron speed. σ may be viewed as a measure of the likelihood, or the probability in the non-mathematical sense, of a given reaction occurring. Alternatively, because σ has the units of meters squared, it is also thought of as the 'effective area' presented to the incident neutron by the target nucleus. Hence σ is called the microscopic cross-section for the neutron reaction but it must be emphasized that it is not the physical area of the nucleus (σ

can have values orders of magnitude larger or smaller than the physical area of the atom. The units for the microscopic cross-section if the **Barn.** The **Barn** is defined by 1 barn = 10^{-28} meters$^{2.}$

Since neutrons can have more than one type of reaction the total microscopic cross-section, the likelihood of the neutron interacting with the nucleus, can be expressed as the sum of a number of partial cross-sections.

B. Macroscopic Cross-Sections

When dealing with matter in bulk, the reaction rate $N\sigma\phi$ is often written as: $\Sigma\phi$ where $\Sigma = N\sigma$ and is known as the macroscopic cross-section. Σ (Capital Sigma) may be regarded as the total collision area presented by the target nuclei per unit volume of material. As it has the units of (area/volume), or $\mathbf{m^{-1}}$ **,** Σ may be interpreted as the probability per meter of tract length that a neutron will interact with the material.

Now the average distance that a neutron travels without interacting is known as the **Mean Free Path.** The mean free path is usually denoted by the Greek letter **Lambda:** λ. The mean free path is equal to the reciprocal of the macroscopic cross-section.

The difference between the microscopic and macroscopic cross sections is extremely important and is restated for clarity. The **microscopic cross section** (σ) represents the effective target area that a single nucleus presents to a bombarding particle. The units are given in **barns** or $\mathbf{cm^2}$. The **macroscopic cross section** (Σ) represents the effective target area that is presented by all of the nuclei contained in $\mathbf{1\ cm^3}$ of the material.

2.4 Nuclear Reaction

A neutron interacts with an atom of the material it enters in two basic ways:

- It will interact through a **scattering interaction**
- or through an **absorption reaction**.

The probability of a neutron being absorbed by a particular atom is the microscopic cross section for absorption, σ_a. The probability of a neutron scattering off of a particular nucleus is the microscopic cross section for scattering, σ_s. The sum of the microscopic cross section for absorption and the microscopic cross section for scattering is the total microscopic cross section:

$$\sigma_T = \sigma_a + \sigma_s$$

where:

σ_T = total microscopic cross section
σ_a = microscopic cross section for absorption
σ_s = microscopic cross section for scattering

Both the absorption and the scattering microscopic cross sections can be further divided. For instance, the scattering cross section is the sum of the elastic scattering cross section (σ_{se}) and the inelastic scattering cross section (σ_{si}).

$$\sigma_s = \sigma_{si} + \sigma_{se}$$

The microscopic absorption cross section (σ_a) includes all reactions except scattering. However, for most purposes it is sufficient to merely separate it into two categories, fission (σ_{af}) and capture (σ_{ac}).

$$\sigma_a = \sigma_{af} + \sigma_{ac}$$

The variation of absorption cross sections with neutron energy is often complicated. For many elements the absorption cross sections are small, ranging from a fraction of a barn to a few barns for slow (or thermal) neutrons.

For a considerable number of nuclides of moderately high (or high) mass numbers, an examination of the variation of the absorption cross section with the energy of the incident neutron reveals the existence of three regions on a curve of absorption cross section versus neutron energy.

A. First, the cross section decreases steadily with increasing neutron energy in a low energy region, which includes the thermal range (**E < 1 eV**). In this region the absorption cross section, which is often high, is inversely proportional to the **velocity** (**v**). This region is frequently referred to as the "1/v region," because the **absorption cross section** is proportional to **1/v,** which is the reciprocal of neutron velocity.

B. Following the 1/v region, there occurs the "**resonance region**" in which the cross sections rise sharply to high values called "resonance peaks" for neutrons of certain energies, and then fall again. These energies are called resonance energies and are a result of the affinity of the nucleus for neutrons whose energies closely match its discrete, quantum energy levels. That is, when the binding energy of a neutron plus the kinetic energy of the neutron are exactly equal to the amount required to raise a compound nucleus from its ground state to a quantum level, resonance absorption occurs. The typical heavy nucleus will have many closely spaced resonances starting in the low energy (eV) range. This is because heavy nuclei are complex and have more possible configurations and corresponding energy states. Light nuclei, being less complex, have fewer possible energy states and fewer resonances that are sparsely distributed at higher energy levels.

C. For higher neutron energies, the absorption cross section steadily decreases as the energy of the neutron increases. This is called the "fast neutron region." In this region the absorption cross sections are usually less than 10 barns. With the exception of hydrogen, for which the value is fairly large, the elastic scattering cross sections are generally small, for

example, 5 barns to 10 barns. This is close to the magnitude of the actual geometric cross sectional area expected for atomic nuclei. In potential scattering, the cross section is essentially constant and independent of neutron energy. Resonance elastic scattering and inelastic scattering exhibit resonance peaks similar to those associated with absorption cross sections. The resonances occur at lower energies for heavy nuclei than for light nuclei. In general, the variations in scattering cross sections are very small when compared to the variations that occur in absorption cross sections.

2.5 Mean Free Path

If a neutron has a certain probability of undergoing a particular interaction in one centimeter of travel, then the inverse of this value describes how far the neutron will travel (in the average case) before undergoing an interaction. This average distance traveled by a neutron before interaction is known as the **mean free path** for that interaction and is represented by the symbol λ. The relationship between the mean free path (λ) and the macroscopic cross section (Σ) is shown below.

$$\Sigma = \frac{1}{\lambda}$$

2.6 Calculation of Macroscopic Cross Section and Mean Free Path

Most materials are composed of several elements, and because most elements are composed of several isotopes, most materials involve many cross sections, one for each isotope involved. Therefore, to include all the isotopes within a given material, it is necessary to determine the macroscopic cross section for each isotope and then sum all the individual macroscopic cross sections.

2.7 Effects of Temperature on Cross Section

The microscopic absorption cross section varies significantly as neutron energy varies. The microscopic cross sections provided on most charts and tables are measured for a standard neutron velocity of **2200 meters/second,** which corresponds to an ambient temperature of **68ºF**. Therefore, if our material is at a higher temperature, the absorption cross section will be lower than the value for **68ºF.**

2.8 Neutron Flux

Macroscopic cross sections for neutron reactions with materials determine the probability of one neutron undergoing a specific reaction per centimeter of travel through that material. If one wants to determine how many reactions will actually occur, it is necessary to know how many neutrons are traveling through the material and how many centimeters they travel each second. It is

convenient to consider the number of neutrons existing in one cubic centimeter at any one instant and the total distance they travel each second while in that cubic centimeter. The number of neutrons existing in a **cm^3** of material at any instant is called **neutron density** and is represented by the symbol **n** with units of **neutrons/cm^3**. The total distance these neutrons can travel each second will be determined by their velocity.

A good way of defining **neutron flux (ϕ)** is to consider it to be the total path length covered by all neutrons in one cubic centimeter during one second. Mathematically, this is the equation below:

$$\phi = n v$$

where:

ϕ = neutron flux **(**neutrons/cm^2-sec)
n = neutron density (neutrons/cm^3)
v = neutron velocity (cm/sec)

The term neutron flux in some applications (for example, cross section measurement) is used as parallel beams of neutrons traveling in a single direction. The **intensity (I)** of a neutron beam is the product of the neutron density times the average neutron velocity. The directional beam intensity is equal to the number of neutrons per unit area and time (neutrons/cm^2-sec) falling on a surface perpendicular to the direction of the beam.

One can think of the neutron flux in a reactor as being comprised of many neutron beams traveling in various directions. Then, the neutron flux becomes the scalar sum of these directional flux intensities. Since the atoms in a reactor do not interact preferentially with neutrons from any particular direction, all of these directional beams contribute to the total rate of reaction. In reality, at a given point within a reactor, neutrons will be traveling in all directions.

2.9 Self-Shielding

In some locations within the reactor, the flux level may be significantly lower than in other areas due to a phenomenon referred to as **neutron shadowing** or **self-shielding**. For example, the interior of a fuel pin or pellet will "see" a lower average flux level than the outer surfaces since an appreciable fraction of the neutrons will have been absorbed and therefore cannot reach the interior of the fuel pin. This is especially important at resonance energies, where the absorption cross sections are large.

SECTION THREE: REACTION RATES

It is possible to determine the rate at which a nuclear reaction will take place based on the neutron flux, cross section for the interaction, and atom density of the target. This relationship illustrates how a change in one of these items affects the reaction rate.

3.1 Reaction Rates

If the total path length of all the neutrons in a cubic centimeter in a second is known, (**neutron flux** (ϕ)), and if the probability of having an interaction per centimeter path length is also known (**macroscopic cross section** (Σ)), multiply them together to get the number of interactions taking place in that cubic centimeter in one second. This value is known as the **reaction rate** and is denoted by the symbol **R.** The reaction rate can be calculated by the equation shown below:

$$\mathbf{R} = \phi \mathbf{x} \Sigma$$

where:

R = reaction rate (reactions/sec)
ϕ = neutron flux (neutrons/cm^2-sec)
Σ = macroscopic cross section (cm^{-1})

Substituting the fact that:

$$\Sigma = \mathbf{N} \mathbf{x} \sigma$$

where:

Σ = macroscopic cross section (cm^{-1})
N = atom density (atoms/cm^3)
σ = microscopic cross section (cm^2)

we get:

$$\mathbf{R} = \phi \mathbf{x} \mathbf{N} \mathbf{x} \sigma$$

The **reaction rate** calculated will depend on which macroscopic cross section is used in the calculation. Normally, the **reaction rate** of greatest interest is the fission reaction rate.

3.2 Reactor Power Calculation

Multiplying the reaction rate per unit volume by the total volume of the core results in the total number of reactions occurring in the core per unit time. If the amount of energy involved in each reaction were known, it would be possible to determine the rate of energy release (power) due to a certain reaction.

In a reactor where the average energy per fission is 200 MeV, it is possible to determine the number of fissions per second that are necessary to produce one watt of power using the following conversion factors.

1 fission = 200 MeV
1 MeV = 1.602 x 10^{-6} ergs
1 erg = 1 x 10^{-7} watt-sec

This is equivalent to stating that 3.12 x 10^{10} fissions release 1 watt-second of energy. The power released in a reactor can be calculated based upon this equation. Multiplying the reaction rate by the volume of the reactor results in the total fission rate for the entire reactor. Dividing by the number of fissions

per watt-sec results in the power released by fission in the reactor in units of watts.

This relationship is shown mathematically in the equation below:

$$P = \frac{\phi_{th}\,\Sigma_f\,V}{3.12 \times 10^{10}}$$

where:

P = power (watts)
ϕ_{th} = thermal neutron flux (neutrons/cm -sec)
Σ_f= macroscopic cross section for fission (**cm^{-1}**)
V = volume of core (cm3)

3.3 Relationship Between Neutron Flux and Reactor Power

In an operating reactor the volume of the reactor is constant. Over a relatively short period of time (days or weeks), the number density of the fuel atoms is also relatively constant. Since the atom density and microscopic cross section are constant, the macroscopic cross section must also be constant. Examining the power equation above it is apparent that if the reactor volume and macroscopic cross section are constant, then the reactor power and the neutron flux are directly proportional.

This is true for day-to-day operation. The neutron flux for a given power level will increase very slowly over a period of months due to the **burnup** of the fuel and resulting decrease in atom density and macroscopic cross section.

SECTION FOUR: NEUTRON MODERATION

In thermal reactors, the neutrons that cause fission are at a much lower energy than the energy level at which they were born from fission. In this type of reactor, specific materials must be included in the reactor design to reduce the energy level of the neutrons in an efficient manner.

4.1 Neutron Slowing Down and Thermalization

Fission neutrons are produced at an average energy level of 2 MeV and immediately begin to slow down as the result of numerous scattering reactions with a variety of target nuclei. After a number of collisions with nuclei, the speed of a neutron is reduced to such an extent that it has approximately the same average kinetic energy as the atoms (or molecules) of the medium in which the neutron is undergoing elastic scattering.

This energy, which is only a small fraction of an electron volt at ordinary temperatures (0.025 eV at 20(C), is frequently referred to as the thermal energy, since it depends upon the temperature. Neutrons whose energies have been reduced to values in this region (< 1 eV) are designated **thermal neutrons**. The process of reducing the energy of a neutron to the thermal region by elastic scattering is referred to as **thermalization,** slowing down, or

moderation. The material used for the purpose of thermalizing neutrons is called a **moderator.**

A good moderator reduces the speed of neutrons in a small number of collisions, but does not absorb them to any great extent. Slowing the neutrons in as few collisions as possible is desirable in order to reduce the amount of neutron leakage from the core and also to reduce the number of resonance absorptions in non-fuel materials. The ideal moderating material (**moderato**r) should have the following nuclear properties.

- large scattering cross section
- small absorption cross section
- large energy loss per collision

A convenient measure of energy loss per collision is the logarithmic energy decrement. The **average logarithmic energy decrement** is the average decrease per collision in the logarithm of the neutron energy. This quantity is represented by the symbol ξ.

$$\xi = \ln E_i - \ln E_f$$

where:

ξ = average logarithmic energy decrement
E_i = average initial neutron energy
E_f = average final neutron energy

The symbol ξ is commonly called the average logarithmic energy decrement because of the fact that a neutron loses, on the average, a fixed fraction of its energy per scattering collision. Since the fraction of energy retained by a neutron in a single elastic collision is a constant for a given material, is also a constant. Because it is a constant for each type of material and does not depend upon the initial neutron energy, is a convenient quantity for assessing the moderating ability of a material. The values for the lighter nuclei are tabulated in a variety of sources.

Since ξ represents the average logarithmic energy loss per collision, the total number of collisions necessary for a neutron to lose a given amount of energy may be determined by dividing ξ into the difference of the natural logarithms of the energy range in question. The number of collisions (**N)** to travel from any energy, E_{high}, to any lower energy, E_{low}, can be calculated as shown below.

$$N = \frac{\ln E_i - \ln E_f}{\xi}$$

or

$$N = \frac{\ln (E_i / E_f)}{\xi}$$

4.2 Macroscopic Slowing Down Power

Although the logarithmic energy decrement is a convenient measure of the ability of a material to slow neutrons, it does not measure all necessary properties of a moderator. A better measure of the capabilities of a material is the macroscopic slowing down power. The **macroscopic slowing down power** (**MSDP**) is the product of the **logarithmic energy decrement** and the **macroscopic cross section for scattering** in the material.

4.3 Moderating Ratio

Macroscopic slowing down power indicates how rapidly a neutron will slow down in the material in question, but it still does not fully explain the effectiveness of the material as a moderator. An element such as boron has a high logarithmic energy decrement and a good slowing down power, but it is a poor moderator because of its high probability of absorbing neutrons. The most complete measure of the effectiveness of a moderator is the moderating ratio. The **moderating ratio** is the ratio of the macroscopic slowing down power to the macroscopic cross section for absorption. The higher the moderating ratio, the more effectively the material performs as a moderator.

$$MR = \frac{\xi \Sigma_S}{\Sigma_a}$$

SECTION FIVE: PROMPT AND DELAYED NEUTRONS

Not all neutrons are released at the same time following fission. Most neutrons are released virtually instantaneously and are called prompt neutrons. A very small fraction of neutrons are released after the decay of fission products and are called delayed neutrons. Although delayed neutrons are a very small fraction of the total number of neutrons, they play an extremely important role in the control of the reactor.

5.1 Neutron Classification

The great majority (over 99%) of the neutrons produced in fission are released within about $\mathbf{10^{-13}}$ seconds of the actual fission event. These are called **prompt neutrons**. A small portion of fission neutrons are **delayed neutrons**, which are produced for some time after the fission process has taken place. The delayed neutrons are emitted immediately following the first beta decay of a fission fragment known as a delayed neutron precursor. An example of a delayed neutron precursor is bromine-87, shown below.

For most applications, it is convenient to combine the known precursors into groups with appropriately averaged properties. These groups vary somewhat depending on the fissile material in use. The fraction of all neutrons that are produced by each of these precursors is called the delayed neutron fraction for that precursor. The total fraction of all neutrons born as delayed neutrons is called the **delayed neutron fraction** (β). The fraction of delayed neutrons produced varies depending on the predominant fissile nuclide in use. The delayed neutron fractions (β) for the fissile nuclides of most interest are as

follows: **uranium^{233}** (0.0026), uranium^{235}, (0.0065), uranium-238^{238} (0.0148), and plutonium^{239}(0.0021).

5.2 Neutron Generation Time

The neutron generation time is the time required for neutrons from one generation to cause the fissions that produce the next generation of neutrons. The generation time for prompt neutrons (ℓ^* - pronounced "ell-star") is the total time from birth to rebirth. Three time intervals are involved:

1. The time it takes a fast neutron to slow down to thermal energy,
2. The time the now thermal neutron exists prior to absorption in fuel,
3. The time required for a fissionable nucleus to emit a fast neutron after neutron absorption.

Fast neutrons slow to thermal energies or leak out of the reactor in 10^{-4} seconds to 10^{-6} seconds, depending on the moderator. In water moderated reactors, thermal neutrons tend to exist for about 10^{-4} seconds before they are absorbed.

Fission and fast neutron production following neutron absorption in a fissionable nucleus occurs in about 10^{-13} seconds. Thus, fast reactors have an ℓ^* of about 10^{-6} seconds, while thermal reactors have an ℓ^* of about 10^{-6} seconds + 10^{-4} seconds, which is about 10^{-4} seconds to 10^{-5} seconds.

On the other hand, the average generation time for the six delayed neutron groups is the total time from the birth of the fast neutron to the emission of the delayed neutron. Again, three time intervals are involved:

1. The time it takes a fast neutron to slow down to thermal energy
2. The time the thermal neutron exists prior to absorption
3. The average time from neutron absorption to neutron emission by the six precursor groups. The average time for decay of precursors from **U^{235}** is 12.5 seconds. The other terms in the delayed neutron generation time are insignificant when compared to this value, and the average delayed neutron generation time becomes approximately 12.5 seconds.

   A neutron generation time in the range of 10^{-4} seconds to 10^{-5} seconds or faster could result in very rapid power excursions, and control would not be possible without the dependence upon delayed neutrons to slow down the rate of the reaction. The average generation time, and hence the rate that power can rise, is determined largely by the delayed neutron generation time.

SECTION SIX: NEUTRON FLUX SPECTRUM

The number of neutrons that exist at a given energy level varies. A plot of either the fraction of neutrons or the neutron flux at a given energy versus the energy level is called a **neutron energy spectrum**. The neutron energy spectrum varies widely for different types of reactors.

6.1 Prompt Neutron Energies

The neutrons produced by fission are high energy neutrons, and almost all fission neutrons have energies between 0.1 MeV and 10 MeV. The neutron energy distribution, or spectrum, may best be described by plotting the fraction of neutrons per MeV as a function of neutron energy. The most probable neutron energy is about 0.7 MeV, and the average energy of fission neutrons is about 2 MeV.

6.2 Thermal and Fast Breeder Reactor Neutron Spectra

The spectrum of neutron energies produced by fission varies significantly from the energy spectrum, or flux, existing in a reactor at a given time. The neutron flux spectra of a thermal reactor and a fast breeder reactor are essentially the same. The differences in the curve shapes may be attributed to the neutron moderation or slowing down effects. No attempt is made to thermalize or slow down neutrons in the fast breeder reactor (liquid metal cooled); therefore, an insignificant number of neutrons exist in the thermal range.

For the thermal reactor (water moderated), the spectrum of neutrons in the fast region (> 0.1 MeV) has a shape similar to that for the spectrum of neutrons emitted by the fission process. In the thermal reactor, the flux in the intermediate energy region (1 eV to 0.1 MeV) has approximately a **1/E** dependence. That is, if the **energy (E**) is halved, the flux doubles. This **1/E** dependence is caused by the slowing down process, where elastic collisions remove a constant fraction of the neutron energy per collision (on the average), independent of energy; thus, the neutron loses larger amounts of energy per collision at higher energies than at lower energies. The fact that the neutrons lose a constant fraction of energy per collision causes the neutrons to tend to "pile up" at lower energies, that is, a greater number of neutrons exist at the lower energies as a result of this behavior.

In the thermal region the neutrons achieve a thermal equilibrium with the atoms of the moderator material. In any given collision they may gain or lose energy, and over successive collisions will gain as much energy as they lose. These thermal neutrons, even at a specific temperature, do not all have the same energy or velocity; there is a distribution of energies, usually referred to as the Maxwell distribution. The energies of most thermal neutrons lie close to the most probable energy, but there is a spread of neutrons above and below this value.

6.3 Most Probable Neutron Velocities

Neutrons in thermal equilibrium with the surrounding material are referred to as **Thermal Neutrons**. It may be shown, through the use of the kinetic theory of perfect gases, that the average kinetic energy of a gas particle in thermal equilibrium, at a temperature **T** is given by:

$$\tfrac{1}{2}\, mV^2 = 3/2\, kT$$

which gives us ½ kT of energy per degree of freedom. In this equation **m** is the mass of the neutron and **k** is the Boltzmann's constant. (1.38×10^{-23} J/K).

Nuclear data are often given for the 'standard' temperature of 293K (20°C). Calculations for thermal neutrons in surroundings at a temperature of 293K gives a mean velocity of 2200 meters per second and corresponds to a kinetic energy of **0.025 eV.** Thus for Uranium 235 and for thermal neutrons at the 'standard' 2200 meters per second we can calculate a macroscopic cross-section of: **690 Barns.**

Sometime the term 'slow neutron' is used as if synonymous with 'thermal neutron' – this is not the case. Also, thermal neutrons are sometimes regarded as being necessarily at 20°C – again this is not the case. For example, neutrons at, say, 250°C would be thermal neutrons with corresponding speed and energy value of (2940 meters/second) and (0.145 eV).

The **most probable velocity** (v_p) of a thermal neutron is determined by the temperature of the medium and can be determined by the following Equation

$$v_p = \sqrt{\frac{3kT}{m}}$$

where:

v_p = most probable velocity of neutron (cm/sec)
k = Boltzman's constant (1.38×10^{-16} erg/ K)
T = absolute temperature in degrees Kelvin (K)
m = mass of neutron (1.66×10^{-24} grams)

From these calculations it is evident that the most probable velocity of a thermal neutron increases as temperature increases. The most probable velocity at 20 C is of particular importance since reference data, such as nuclear cross sections, are tabulated for a neutron velocity of 2200 meters per second.

END OF CHAPTER 2-NEUTRON PHYSICS

CHAPTER THREE: INTRODUCTION TO REACTOR PHYSICS

SECTION ONE: NEUTRON LIFE CYCLE

Some number of the fast neutrons produced by fission in one generation will eventually cause fission in the next generation. The series of steps that fission neutrons go through as they slow to thermal energies and are absorbed in the reactor is referred to as the neutron life cycle. The neutron life cycle is markedly different between fast reactors and thermal reactors. This section of this white paper presents the neutron life cycle for thermal reactors.

1.1 The Chain Reaction

Although the binary fission of a nucleus by an impinging neutron would no doubt be of academic interest, it would in itself be of little significance in the context of large scale power production. It is the accompanying release of neutrons which makes power production possible in that the neutrons may induce further nuclei to fission and thus lead to a chain reaction.

1.2 Infinite Multiplication Factor, k_∞

Not all of the neutrons produced by fission will have the opportunity to cause new fissions because some neutrons will be absorbed by non-fissionable material. Some will be absorbed parasitically in fissionable material and will not cause fission, and others will leak out of the reactor.

For the maintenance of a self-sustaining chain reaction, however, it is not necessary that every neutron produced in fission initiate another fission. The minimum condition is for each nucleus undergoing fission to produce, on the average, at least one neutron that causes fission of another nucleus. This condition is conveniently expressed in terms of a multiplication factor.

The number of neutrons absorbed or leaking out of the reactor will determine the value of this multiplication factor, and will also determine whether a new generation of neutrons is larger, smaller, or the same size as the preceding generation. Any reactor of a finite size will have neutrons leak out of it.

Generally, the larger the reactor, the lower the fraction of neutron leakage. For simplicity, we will first consider a reactor that is infinitely large, and therefore has no neutron leakage. A measure of the increase or decrease in neutron flux in an infinite reactor is the infinite multiplication factor, **k_∞.** The **infinite multiplication factor** is the ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption in the preceding generation. This can be expressed mathematically as shown below:

$$k_\infty = \frac{\text{Neutron Production from Fission in one Generation}}{\text{neutron absorption in the preceding generation}}$$

For simplicity, at the beginning of our discussion, it is advantageous to consider the presupposition that there are no losses of neutrons out of the system; that is, the system – the nuclear reactor – is assumed to be of infinite size. The symbol $\mathbf{k_\infty}$ is now used for the multiplication constant, the subscript infinity (∞) being a reminder of the imposed condition of an infinite sized nuclear reactor. From our definition of $\mathbf{k_\infty}$, the multiplication constant it is clear that:

- If $\mathbf{k_\infty} > 1$ the chain reaction is divergent and the nuclear reactor is said to be supercritical. As the power produced is related to the number of fission events taking place, this is the condition for power raising. The larger the value of $\mathbf{k_\infty}$ the quicker is the rate of power rise.

- If $\mathbf{k_\infty} = 1$ the chain reaction is self-sustained and the reactor is said to be critical. A nuclear power station producing power at a steady rate for perhaps months or years – a base load power station – is therefore a critical reactor. The word is unfortunate, but understandable when it is recalled that the term 'critical' dates back to 12 December 1942. On that day Enrico Fermi and his colleagues first achieved a self-sustained chain reaction, and created the first nuclear pile.

- If $\mathbf{k_\infty} < 1$ the chain reaction is converging and the reactor power is decreasing. As before the rate of power decrease is determined by how much less than unity is the value of $\mathbf{k_\infty}$.

1.3 Chain Reaction in Natural Uranium – Fast Neutrons

Let us first consider an infinite mass of natural uranium, representing the simplest conceptual design for a nuclear reactor. Assume $\mathbf{n_1}$ fast neutrons (i.e., with energy of fission neutrons $\cong$ 2 MeV) are introduced into the reactor. To determine the value of $\mathbf{k_\infty}$ for the reactor it is necessary to calculate the number of neutrons in the generation following the absorption in the uranium of all the original generation of $\mathbf{n_1}$ fast neutrons.

In considering the possible fate of the $\mathbf{n_1}$ neutrons it is important to recall that natural uranium consists overwhelmingly of $\mathbf{U^{338}}$. There are three possibilities.

A. Fission

Fast neutrons can induce fission in $\mathbf{U^{238}}$ provided they have energy in excess of the threshold 1.1 MeV, the fission cross-section being much the same value as for $\mathbf{U^{235}}$. Thus the fission event taking place in the reactor can be regarded as being predominantly of $\mathbf{U^{238}}$ nuclei and giving rise to the next generation of fast neutrons, $\mathbf{n_2}$ say.

B. Capture

Some of the $\mathbf{n_1}$ neutrons will undergo direct capture in the natural uranium, i.e., non-fission absorptions in the $\mathbf{U^{238}}$ and $\mathbf{U^{235}}$.

C. Scatter

Some of the $\mathbf{n_1}$ neutrons will be scattered but only those undergoing inelastic scattering are significant. In this case the neutrons will emerge from the inelastic scattering event with energy less than the threshold value of 1.1 MeV necessary for $\mathbf{U^{238}}$ fission and will subsequently be captured in the resonance capture peaks of the $\mathbf{U^{238}}$. In this context therefore inelastic scattering may be regarded as leading indirectly to capture.

Elastic scattering events may be ignored because the neutrons are effectively unchanged by the collision and are still identified as being of the original $\mathbf{n_1}$ neutron generation.

1.4. k_∞ for Natural Uranium and Fast Neutrons

The initial $\mathbf{n_1}$ neutrons will therefore either cause fission or be captured (directly or indirectly via inelastic scattering collision). The fraction of causing fission may be calculated using the cross-section values for the nuclear reactions and hence the next generation of neutrons, $\mathbf{n_2}$, may be determined analytically . This solution for $\mathbf{n_2}$, yields $\mathbf{n_2} = 0.26\mathbf{n_1}$

Therefore $\mathbf{k_\infty} = 0.26 < 1$. Hence natural Uranium, no matter what the geometry cannot sustain a chain reaction.

1.5. To Achieve $k_\infty > 1$

The value of is determined by the balance between neutron **Production** in the reactor fuel and neutron loss by **Absorption** in the reactor materials and by **Leakage** out of the reactor for a finite system. In this paper we have discussed the impossibility of having a chain reaction in natural uranium. There are two ways in which we can increase the value of $\mathbf{k_\infty}$.

1. Change the properties of the fuel
2. Change the properties of the impinging neutrons.

A. Fuel Enrichment

$\mathbf{k_\infty}$ is less than unity, in our previous discussion, largely because the inelastic collisions of the fast neutrons in the natural uranium fuel leads to the subsequent loss of the neutrons in the $\mathbf{U^{238}}$ resonance capture peaks. The fast fission cross-section of $\mathbf{U^{238}}$ and $\mathbf{U^{235}}$ are much the same and too low to counter the neutron losses.

It follows that if the proportion of $\mathbf{U^{238}}$ in the fuel is reduced, thus increasing the proportion of $\mathbf{U^{235}}$, the effect is to replace a high neutron capture material $\mathbf{U^{238}}$ by another material $\mathbf{U^{235}}$ which has much the same

fast fission cross-section but less neutron capture characteristics. This process of increasing the proportion of U^{235} is known as **Fuel Enrichment.**

As U^{235} has no energy threshold for fission, the net result of fuel enrichment is that the neutron from fission increases and neutron absorption decreases. Calculation shows that U^{235} enrichment of at least 20% is necessary to give $k_\infty > 1$ for fast neutrons. This enrichment process forms the basis of 'fast reactors', the title fast reactor being derived from the fact that the neutrons inducing fission are largely energetic fast neutrons.

B. Neutron Moderation

The alternative to large fuel enrichment and fast neutrons is to retain, slightly enriched natural uranium as the fuel but provide the nuclear chain reaction with slow neutrons, and thus take advantage of the greatly increased likelihood of U^{235} fission occurring.

Instead of having n_1 fast neutrons, we consider the chain reaction to consist of n_1 **slow** neutrons introduced into our infinite mass of nuclear material. We then repeat our calculations to obtain the next generation of n_2 neutrons. In this instance there is no possibility with slow neutrons of inelastic collisions occurring. The neutrons are simply either captured or cause fission.

1.6. Four Factor Formula

A group of fast neutrons produced by fission can enter into several reactions. Some of these reactions reduce the size of the neutron group while other reactions allow the group to increase in size or produce a second generation. There are four factors that are completely independent of the size and shape of the reactor that give the inherent multiplication ability of the fuel and moderator materials without regard to leakage. This **four factor formula** accurately represents the infinite multiplication factor as shown in the equation below.

$$k_\infty = \varepsilon\, p\, f\, \eta$$

where:

ε = fast fission factor
p = resonance escape probability
f = thermal utilization factor
η = reproduction factor

Each of these four factors, which are explained in the following subsections, represents a process that adds to or subtracts from the initial neutron group produced in a generation by fission.

A. Fast Fission Factor, (ε)

The first process that the neutrons of one generation may undergo is **fast fission. Fast fission** is fission caused by neutrons that are in the fast energy range. Fast fission results in the net increase in the fast neutron population of the reactor core. The cross section for fast fission in **uranium-235, U^{235}** or **uranium-238, U^{238}** is small; therefore, only a small number of fast neutrons cause fission.

The fast neutron population in one generation is therefore increased by a factor called the fast fission factor. **The fast fission factor** (ε) is defined as the ratio of the net number of fast neutrons produced by all fissions to the number of fast neutrons produced by thermal fissions.

The mathematical expression of this ratio is shown below.

$$\varepsilon = \frac{\text{number of fast neutrons produced by all fission}}{\text{number of fast neutrons produced by thermal fissions}}$$

In order for a neutron to be absorbed by a fuel nucleus as a fast neutron, it must pass close enough to a fuel nucleus while it is a fast neutron. The value of ε will be affected by the arrangement and concentrations of the fuel and the moderator. The value of is essentially 1.00 for a homogenous reactor where the fuel atoms are surrounded by moderator atoms.

However, in a heterogeneous reactor, all the fuel atoms are packed closely together in elements such as pins, rods, or pellets. Neutrons emitted from the fission of one fuel atom have a very good chance of passing near another fuel atom before slowing down significantly. The arrangement of the core elements results in a value of about 1.03 for in most heterogeneous reactors. The value of ε is not significantly affected by variables such as temperature, pressure, enrichment, or **neutron poison** concentrations. **Poison**s are non-fuel materials that easily absorb neutrons and will be discussed in more detail later.

B. Resonance Escape Probability, (p)

After increasing in number as a result of some fast fissions, the neutrons continue to diffuse through the reactor. As the neutrons move they collide with nuclei of fuel and non-fuel material and moderator in the reactor losing part of their energy in each collision and slowing down.

While they are slowing down through the resonance region of uranium-238, (**U^{238}**) which extends from about 6 eV to 200 eV, there is a chance that some neutrons will be captured. The probability that a neutron will not be absorbed by a resonance peak is called the **resonance escape probability**. The **resonance escape probability** (**p**) is defined as the

ratio of the number of neutrons that reach thermal energies to the number of fast neutrons that start to slow down. This ratio is shown below.

$$p = \frac{\text{Number of nuetonrs that reach thermal energy}}{\text{number of fast neutrons that start to slow down}}$$

The value of the resonance escape probability is determined largely by the fuel-moderator arrangement and the amount of enrichment of uranium-235 ($\mathbf{U^{235}}$). To undergo resonance absorption, a neutron must pass close enough to a uranium-238 ($\mathbf{U^{238}}$) nucleus to be absorbed while slowing down.

In a homogeneous reactor the neutron does its slowing down in the region of the fuel nuclei, and this condition is easily met. This means that a neutron has a high probability of being absorbed by uranium-238 ($\mathbf{U^{238}}$) while slowing down; therefore, its escape probability is lower. In a heterogeneous reactor, however, the neutron slows down in the moderator where there are no atoms of uranium-238 ($\mathbf{U^{238}}$) present. Therefore, it has a low probability of undergoing resonance absorption, and its escape probability is higher.

The value of the resonance escape probability is not significantly affected by pressure or poison concentration. In water moderated, low uranium-235 ($\mathbf{U^{235}}$) enrichment reactors, raising the temperature of the fuel will raise the resonance absorption in uranium-238 ($\mathbf{U^{238}}$) due to the Doppler effect (an apparent broadening of the normally narrow resonance peaks due to thermal motion of nuclei). The increase in resonance absorption lowers the resonance escape probability, and the fuel temperature coefficient for resonance escape is negative. The temperature coefficient of resonance escape probability for the moderator temperature is also negative. The decrease in water density, as temperature increases, allows more resonance energy neutrons to enter the fuel and be absorbed. The value of the resonance escape probability is always less than one. The product of the fast fission factor and the resonance escape probability (p) is the ratio of the number of fast neutrons that survive thermalization compared to the number of fast neutrons originally starting the generation.

C. Thermal Utilization Factor, (f)

Once thermalized, the neutrons continue to diffuse throughout the reactor and are subject to absorption by other materials in the reactor as well as the fuel. The thermal utilization factor describes how effectively thermal neutrons are absorbed by the fuel, or how well they are utilized within the reactor. The **thermal utilization factor** (**f**) is defined as the ratio of the number of thermal neutrons absorbed in the fuel to the number of thermal neutrons absorbed in any reactor material. This ratio is shown below.

$$f = \frac{\text{number of thermal neutrons absorbed in the fuel}}{\text{number of thermal neutrons absorbed in all reactor materials}}$$

The thermal utilization factor will always be less than one because some of the thermal neutrons absorbed within the reactor will be absorbed by atoms of non-fuel materials.

An equation can be developed for the thermal utilization factor in terms of reaction rates as follows:

$$f = \frac{\text{number of thermal neutrons absorbed in the fuel}}{\text{number of thermal neutrons absorbed in all reactor materials}}$$

$$f = \frac{\Sigma_a^u \phi^u v^u}{\Sigma_a^u \phi^u v^u + \Sigma_a^m \phi^m v^m + \Sigma_a^p \phi^p v^p}$$

Σ = macroscopic cross section (cm^{-1})
ϕ = neutron flux (neutrons/cm^2-sec)
n = neutron density (neutrons/cm^3)
v = neutron velocity (cm/sec)

The superscripts u, m, and p refer to uranium, moderator, and poison, respectively.

In a heterogeneous reactor, the flux will be different in the fuel region than in the moderator region due to the high absorption rate by the fuel. Also, the volumes of fuel, moderator, and poisons will be different. Although not shown in the above equation, other non-fuel materials, such as core construction materials, may absorb neutrons in a heterogeneous reactor. These other materials are often lumped together with the superscript designation OS, for "other stuff." To be completely accurate, the above equation for the thermal utilization factor should include all neutron-absorbing reactor materials when dealing with heterogeneous reactors.

In a homogeneous reactor the neutron flux seen by the fuel, moderator, and poisons will be the same. Also, since they are spread throughout the reactor, they all occupy the same volume. This allows the previous equation to be rewritten as shown below:

$$f = \frac{\Sigma_a^u}{\Sigma_a^u + \Sigma_a^m + \Sigma_a^p}$$

The equation above gives an approximation for a heterogeneous reactor if the fuel and moderator are composed of small elements distributed uniformly throughout the reactor.

Since absorption cross sections vary with temperature, it would appear that the thermal utilization factor would vary with a temperature change. But, substitution of the temperature correction formulas in the above equation will reveal that all terms change by the same amount, and the ratio remains the same. In heterogeneous water-moderated reactors, there is another important factor. When the temperature rises, the water moderator expands, and a significant amount of it will be forced out of the

reactor core. This means that the number of moderator atoms per **cm^3**, will be reduced, making it less likely for a neutron to be absorbed by a moderator atom. This reduction in N_m results in an increase in thermal utilization as moderator temperature increases because a neutron now has a better chance of hitting a fuel atom. Because of this effect, the temperature coefficient for the thermal utilization factor is positive.

The amount of enrichment of uranium-235 (**U^{235}**) and the poison concentration will affect the thermal utilization factor in a similar manner as can be seen from the equation above.

D. Reproduction Factor, (η)

Most of the neutrons absorbed in the fuel cause fission, but some do not. **The reproduction factor (η)** is defined as the ratio of the number of fast neutrons produces by thermal fission to the number of thermal neutrons absorbed in the fuel. The reproduction factor is shown below:

$$\eta = \frac{\text{number of fast neutrons produced by thermal fission}}{\text{number of thermal neutrons absorbed in the fuel}}$$

The reproduction factor can also be stated as a ratio of rates as shown below:

$$\eta = \frac{\text{rate of production of fast neutrons by thermal fission}}{\text{rate of absorption of thermal neutrons by the fuel}}$$

The rate of production of fast neutrons by thermal fission can be determined by the product of the fission reaction rate ($\Sigma_f^u \phi^u$) and the average number of neutrons produced per fission (ν). The average number of neutrons released in thermal fission of uranium-235 (**U^{235}**) is 2.42.

The rate of absorption of thermal neutrons by the fuel is $\Sigma_a^u \phi^u$ **.** Substituting these terms into the equation above results in the following equation:

$$\eta = \frac{\Sigma_f^u \phi^u \nu}{\Sigma_a^u \phi^u}$$

η = reproduction factor
Σ = macroscopic cross section (cm^{-1})
ϕ = neutron flux **(**neutrons/cm^2-sec)
ν = average number of neutrons per fission

As temperature varies, each absorption and fission microscopic cross section varies according to the **1/v** relationship. Since both the numerator and the denominator change equally, the net change in h is zero. Therefore, η changes only as uranium-235 (**U^{235}**)enrichment changes. η increases with enrichment because there is less uranium-238 (**U$^{238)}$** in the

reactor making it more likely that a neutron absorbed in the fuel will be absorbed by uranium-235 ($\mathbf{U^{235}}$) and cause fission.

1.7 Effective Multiplication Factor

The infinite multiplication factor can fully represent only a reactor that is infinitely large, because it assumes that no neutrons leak out of the reactor. To completely describe the neutron life cycle in a real, finite reactor, it is necessary to account for neutrons that leak out.

The multiplication factor that takes leakage into account is the **effective multiplication factor (k_{eff})**, which is defined as the ratio of the neutrons produced by fission in one generation to the number of neutrons lost through absorption and leakage in the preceding generation. The effective multiplication factor may be expressed mathematically as shown below:

$$k_{eff} = \frac{\text{neutron production from fission in one reneration}}{\text{neutron absorption in the preceding generation} + \text{neutron leakage in the preceding generation}}$$

So, the value of k_{eff} for a self-sustaining chain reaction of fissions, where the neutron population is neither increasing nor decreasing, is one. The condition where the neutron chain reaction is self-sustaining and the neutron population is neither increasing nor decreasing is referred to as the **critical condition** and can be expressed by the simple equation:

$$k_{eff} = 1 .$$

If the neutron production is greater than the absorption and leakage, the reactor is called supercritical. In a **supercritical reactor**, k_{eff} is greater than one, and the neutron flux increases in each generation.

If, on the other hand, the neutron production is less than the absorption and leakage, the reactor is called **subcritical.** In a **subcritical reactor**, k_{eff} is less than one, and the flux decreases each generation.

When the multiplication factor of a reactor is not equal to exactly one, the neutron flux will change and cause a change in the power level. Therefore, it is essential to know more about how this factor depends upon the contents and construction of the reactor. The balance between production of neutrons and their absorption in the core and leakage out of the core determines the value of the multiplication factor.

If the leakage is small enough to be neglected, the multiplication factor depends upon only the balance between production and absorption, and is called the **infinite multiplication factor k_∞** since an infinitely large core can have no leakage.
When the leakage is included, the factor is called the effective multiplication factor k_{eff}. The effective multiplication factor k_{eff} for a finite reactor may be

expressed mathematically in terms of the infinite multiplication factor and two additional factors which account for neutron leakage as shown below:

$$k_{eff} = k_{\infty} \times \mathscr{L}_f \times \mathscr{L}_t$$

where:

k_{eff} = effective multiplication factor
k_{∞} = infinite multiplication factor
$\mathscr{L}_f$ = fast non-leakage probability
$\mathscr{L}_t$ = thermal non-leakage probability

A. Fast Non-Leakage Probability ($\mathscr{L}_f$)

In a realistic reactor of finite size, some of the fast neutrons leak out of the boundaries of the reactor core before they begin the slowing down process. The **fast non-leakage probability** ($\mathscr{L}_f$) is defined as the ratio of the number of fast neutrons that do not leak from the reactor core to the number of fast neutrons produced by all fissions. This ratio is stated as follows:

$$\mathscr{L}_f = \frac{\text{number of fast neutrons that do not leak from reactor}}{\text{number of fast neutrons produced by all fissions}}$$

B. Thermal Non-Leakage Probability ($\mathscr{L}_t$)

Neutrons can also leak out of a finite reactor core after they reach thermal energies. The **thermal non-leakage probability** ($\mathscr{L}_t$) is defined as the ratio of the number of thermal neutrons that do not leak from the reactor core to the number of neutrons that reach thermal energies. The thermal non-leakage probability is represented by the following equation:

$$\mathscr{L}_t = \frac{\text{number of thermal neutrons that do not leak from reactor}}{\text{number of neutrons that reach thermal energies}}$$

The fast non-leakage probability ($\mathscr{L}_f$) and the thermal non-leakage probability ($\mathscr{L}_t$) may be combined into one term that gives the fraction of all neutrons that do not leak out of the reactor core. This term is called the **total non-leakage probability** and is given the symbol $\mathscr{L}_T$.

where $\mathscr{L}_T = \mathscr{L}_t \times \mathscr{L}_f$, $\mathscr{L}_f$ and $\mathscr{L}_t$ are both effected by a change in coolant temperature in a heterogeneous water-cooled, water-moderated reactor.

As coolant temperature rises, the coolant expands. The density of the moderator is lower; therefore, neutrons must travel farther while slowing down. This effect increases the probability of leakage and thus decreases the non-leakage probability. Consequently, the temperature coefficient (defined later) for the non-leakage probabilities is negative, because as temperature rises, $\mathscr{L}_f$ and $\mathscr{L}_t$ decrease.

1.8 Six Factor Formula

With the inclusion of these last two factors it is possible to determine the fraction of neutrons that remain after every possible process in a nuclear reactor. The effective multiplication factor ($\mathbf{k_{ef}}$) can then be determined by the product of six terms.

$$\mathbf{k_{eff}} = \varepsilon \mathscr{L}_f \mathbf{p} \mathscr{L}_t \mathbf{f} \eta$$

where:

$\mathbf{k_{eff}}$ = effective multiplication factor
ε = fast fission factor
$\mathscr{L}_f$ = fast non-leakage probability
p = resonance escape probability
$\mathscr{L}_t$ = thermal non-leakage probability
f = thermal utilization factor
η = reproduction factor

The equation above is called the **six factor formula**. Using this six factor formula, it is possible to trace the entire neutron life cycle from production by

1.9 Neutron Life Cycle of a Fast Reactor

The neutron life cycle in a fast reactor is markedly different than that for a thermal reactor. In a fast reactor, care is taken during the reactor design to minimize thermalization of neutrons. Virtually all fissions taking place in a fast reactor are caused by fast neutrons. Due to this, many factors that are taken into account by the thermal reactor neutron life cycle are irrelevant to the fast reactor neutron life cycle. The resonance escape probability is not significant because very few neutrons exist at energies where resonance absorption is significant.

The thermal non-leakage probability does not exist because the reactor is designed to avoid the thermalization of neutrons. A separate term to deal with fast fission is not necessary because all fission is fast fission and is handled by the reproduction factor.

The thermal utilization factor is modified to describe the utilization of fast neutrons instead of thermal neutrons. The reproduction factor is similarly modified to account for fast fission instead of thermal fission.

SECTION TWO: REACTIVITY

Reactivity is a measure of the departure of a reactor from criticality. The reactivity is related to the value of $\mathbf{k_{eff}}$. Reactivity is a useful concept to predict how the neutron population of a reactor will change over time.

2.1. Application of the Effective Multiplication Factor

When $\mathbf{k_{eff}}$ remains constant from generation to generation, it is possible to determine the number of neutrons beginning any particular generation by knowing only the value of $\mathbf{k_{eff}}$ and the number of neutrons starting the first generation. If $\mathbf{N_0}$ neutrons start the first generation, then $\mathbf{N_0 \times k_{eff}}$ neutrons

start the second generation. The following equation can be used to calculate the number of neutrons after the completion of "**n**" generations:

$$N_n = N_0 x(k_{eff})^n$$

Where:

N_n = Number of neutrons after the completion of "**n**" generations
N_0 = Number of neutrons in the first generation
k_{eff} = effective multiplication factor
n = Number of generations

2.2. Reactivity

If there are N_0 neutrons in the preceding generation, then there are $N_0 x k_{eff}$ neutrons in the present generation. The numerical change in neutron population is $(N_0 x k_{eff} - N_0)$. The gain or loss in neutron population $(N_0 x k_{eff} - N_0)$, expressed as a fraction of the present generation $(N_0 x k_{eff})$, is shown below:

$$\rho = \frac{N_0 x k_{eff} - N_0}{N_0 x k_{eff}}$$

Where:

ρ = reactivity: The fractional change in neutron population pear generation
N_0 = Number of neutrons in the first generation
k_{eff} = effective multiplication factor

This relationship represents the fractional change in neutron population per generation and is referred to as **reactivity (ρ).** Cancelling out the term N_0 from the numerator and denominator, the reactivity is determined as shown in the equation below:

$$\rho = \frac{k_{eff} - 1}{k_{eff}}$$

Where:

ρ = reactivity:
k_{eff} = effective multiplication factor

From the equation above it may be seen that ρ may be positive, zero, or negative, depending upon the value of k_{eff}. The larger the absolute value of reactivity in the reactor core, the further the reactor is from criticality. It may be convenient to think of reactivity as a measure of a reactor's departure from criticality.

2.3. Units of Reactivity

Reactivity is a dimensionless number. It is simply a ratio of two dimensionless quantities which is often a small decimal value. In order to make this value easier to express, artificial units are defined. By definition, the value for **reactivity** that results directly from the calculation of the equation above is in

units of **Δk/k**. Alternative units for reactivity are **%Δk/k** and **pcm** (percent millirho). The conversions between these units of reactivity are shown below:

$$\frac{\%\Delta k}{k} = \frac{0.01 \times \Delta k}{k}$$

and:

$$pcm = \frac{0.00001 \times \Delta k}{k}$$

Another unit of reactivity that is used at some reactors is equivalent to $\mathbf{10^{-4}}$ **Δk/k.** This unit of reactivity does not have a unique name. Special units for reactivity that do have unique names are dollars and cents. These units and their applications will be described later in this white paper.

2.4. Reactivity Coefficients and Reactivity Defects

The amount of reactivity (ρ) in a reactor core determines what the neutron population, and consequently the reactor power, are doing at any given time. The reactivity can be effected by many factors (for example, fuel depletion, temperature, pressure, or poisons). In this white paper we will discuss the factors affecting reactivity and how they are used to control or predict reactor behavior.

To quantify the effect that a variation in parameter (that is, increase in temperature, control rod insertion, increase in neutron poison) will have on the reactivity of the core, **reactivity coefficients** are used. **Reactivity coefficients** are the amount that the reactivity will change for a given change in the parameter. For instance, an increase in moderator temperature will cause a decrease in the reactivity of the core. The amount of reactivity change per degree change in the moderator temperature is the moderator temperature coefficient. Typical units for the moderator temperature coefficient are **pcm/°F**.

Reactivity coefficients are generally symbolized by α_x, where **x** represents some variable reactor parameter that affects reactivity. The definition of a reactivity coefficient in equation format is shown below:

$$\alpha_x = \frac{\Delta\rho}{\Delta x}$$

If the parameter **x** increases and positive reactivity is added, then α_x is positive. If the parameter **x** increases and negative reactivity is added, then α_x is negative.

Reactivity defects Δρ are the total reactivity change caused by a variation in a parameter. Reactivity defects can be determined by multiplying the change in the parameter by the average value of the reactivity coefficient for that parameter. The equation below shows the general method for relating reactivity coefficients to reactivity defects:

$$\Delta\rho = \alpha_x \Delta x$$

SECTION THREE: REACTIVITY COEFFICIENTS

Changes in the physical properties of the materials in the reactor will result in changes in the reactivity. Reactivity coefficients are useful in quantifying the reactivity change that will occur due to the change in a physical property such as the temperature of the moderator or fuel.

3.1. Moderator Effects

As discussed in other introductory white papers, a moderator possesses specific desirable characteristics:

- Large neutron scattering cross section
- Low neutron absorption cross section
- large neutron energy loss per collision

With the exception of the **Liquid Metal Fast Breeder Reactor** (**LMFBR**), the remaining major reactor types that are currently employed use moderating materials to reduce fission neutron energies to the thermal range. Light moderators (composed of light nuclei) are found to be more effective than heavy moderators because the light moderator removes more energy per collision than a heavy moderator. Therefore, the neutrons reach thermal energy more rapidly and they are less likely to be lost through resonance absorption.

As discussed in other white papers, the ability of a given material to slow down neutrons is referred to as the **macroscopic slowing down power** (**MSDP**) and is defined as the product of the **logarithmic energy decrement per collision** (ξ) times the **macroscopic scattering cross section** for neutrons as follows:

$$\mathbf{MSDP} = \xi \times \Sigma_s$$

Where:

MSDP = macroscopic slowing down power
ξ = logarithmic energy decrement per collision
Σ_s = macroscopic scattering cross section (cm^{-1})

Macroscopic slowing down power indicates how rapidly slowing down occurs in the material in question, but it does not completely define the effectiveness of the material as a moderator. An element such as boron has a high logarithmic energy decrement and a good slowing down power, but is a poor moderator. It is a poor moderator because of its high probability of absorbing neutrons, and may be accounted for by dividing the macroscopic slowing down power by the macroscopic absorption cross section. This relationship is called the **moderating ratio** (**MR**):

$$\mathbf{MR} = \frac{\xi \times \Sigma_s}{\Sigma_a}$$

Where:

MR = moderator ratio

ξ = logarithmic energy decrement per collision
Σ_s = macroscopic scattering cross section (cm^{-1})
Σ_a = macroscopic absorption cross section (cm^{-1})

The moderating ratio is merely the ratio of slowing down power to the **macroscopic absorption cross section**. The higher the **moderating ratio**, the more effectively the material performs as a moderator.

Another ratio, the **moderator-to-fuel ratio** (N^m/N^u), is very important in the discussion of moderators. As the reactor designer increases the amount of moderator in the core (that is, N^m/N^u increases), neutron leakage decreases.

Neutron absorption in the moderator (Σ_a^m) increases and causes a decrease in the thermal utilization factor. Having insufficient moderator in the core (that is, N^m/N^u decreases) causes an increase in slowing down time and results in a greater loss of neutrons by resonance absorption. This also causes an increase in neutron leakage. This effect is beneficial in a thermal reactor in the event of a **LOCA** (Loss of Coolant Accident).

Because the moderator-to-fuel ratio affects the thermal utilization factor and the resonance escape probability, it also affects k_{eff}. The remaining factors in the six factor formula are also affected by the moderator-to-fuel ratio, but to a lesser extent than **f** and **p**.

For a Nuclear Power Plant with a large core fueled with low-enriched fuel, there is an optimum point above which increasing the moderator-to-fuel ratio decreases k_{eff} due to the dominance of the decreasing thermal utilization factor. Below this point, a decrease in the moderator-to-fuel ratio decreases k_{eff} due to the dominance of the increased resonance absorption in the fuel. If the ratio is above this point, the core is said to be over moderated, and if the ratio is below this point, the core is said to be under moderated.

In practice, water-moderated reactors are designed with a moderator-to-fuel ratio so that the reactor is operated in an under moderated condition. The reason that some reactors are designed to be under moderated is if the reactor were over moderated, an increase in temperature would decrease the N^m/N^u due to the expansion of the water as its density became lower. This decrease in N^m/N^u would be a positive reactivity addition, increasing k_{eff} and further raising power and temperature in a dangerous cycle. If the reactor is under moderated, the same increase in temperature results in the addition of negative reactivity, and the reactor becomes more self-regulating.

3.2. Moderator Temperature Coefficient

The change in reactivity per degree change in temperature is called the **temperature coefficient of reactivity**. Because different materials in the reactor have different reactivity changes with temperature and the various

materials are at different temperatures during reactor operation, several different temperature coefficients are used.

Usually, the two dominant temperature coefficients are:

- the moderator temperature coefficient
- the fuel temperature coefficient.

The change in reactivity per degree change in moderator temperature is called the **moderator temperature coefficient** of reactivity. The magnitude and sign (+ or -) of the moderator temperature coefficient is primarily a function of the moderator-to-fuel ratio. If a reactor is under moderated, it will have a negative moderator temperature coefficient. If a reactor is over moderated, it will have a positive moderator temperature coefficient. A negative moderator temperature coefficient is desirable because of its self-regulating effect. For example, an increase in reactivity causes the reactor to produce more power. This raises the temperature of the core and adds negative reactivity, which slows down, or turns, the power rise.

3.3. Fuel Temperature Coefficient

Another temperature coefficient of reactivity, the fuel temperature coefficient, has a greater effect than the moderator temperature coefficient for some reactors. The **fuel temperature coefficient** is the change in reactivity per degree change in fuel temperature. This coefficient is also called the "prompt" temperature coefficient because an increase in reactor power causes an immediate change in fuel temperature.

A negative fuel temperature coefficient is generally considered to be even more important than a negative moderator temperature coefficient because fuel temperature immediately increases following an increase in reactor power. The time for heat to be transferred to the moderator is measured in seconds. In the event of a large positive reactivity insertion, the moderator temperature cannot turn the power rise for several seconds, whereas the fuel temperature coefficient starts adding negative reactivity immediately.

Another name applied to the fuel temperature coefficient of reactivity is the **fuel Doppler reactivity coefficient**. This name is applied because in typical low enrichment, light water-moderated, thermal reactors the fuel temperature coefficient of reactivity is negative and is the result of the **doppler effect**, also called **doppler broadening**. The phenomenon of the Doppler effect is caused by an apparent broadening of the resonances due to thermal motion of nuclei. Stationary nuclei absorb only neutrons of energy $\mathbf{E_0}$. If the nucleus is moving away from the neutron, the velocity (and energy) of the neutron must be greater than $\mathbf{E_0}$ to undergo resonance absorption. Likewise, if the nucleus is moving toward the neutron, the neutron needs less energy than $\mathbf{E_0}$ to be absorbed. Raising the temperature causes the nuclei to vibrate more rapidly within their lattice structures, effectively broadening the energy range of neutrons that may be resonantly absorbed in the fuel. Two nuclides present in

large amounts in the fuel of some reactors with large resonant peaks that dominate the doppler effect are **Uranium-238** (**U^{238}**) and **Plutonium-240** (**Pu^{240}**).

3.4. Pressure Coefficient

The reactivity in a reactor core can be affected by the system pressure. The **pressure coefficient** of reactivity is defined as the change in reactivity per unit change in pressure. The pressure coefficient of reactivity for the reactor is the result of the effect of pressure on the density of the moderator. For this reason, it is sometimes referred to as the moderator density reactivity coefficient.

As pressure increases, density correspondingly increases, which increases the moderator-to-fuel ratio in the core. In the typical under moderated core the increase in the moderator-to-fuel ratio will result in a positive reactivity addition. In reactors that use water as a moderator, the absolute value of the pressure reactivity coefficient is seldom a major factor because it is very small compared to the moderator temperature coefficient of reactivity.

3.5. Void Coefficient

In systems with boiling conditions, such as **boiling water reactors** (**BWR**), the pressure coefficient becomes an important factor due to the larger density changes that occur when the vapor phase of water undergoes a pressure change. Of prime importance during operation of a **BWR**, and a factor in some other water-moderated reactors, is the **void coefficient**. The **void coefficient** is caused by the formation of steam voids in the moderator.

The **void coefficient** of reactivity is defined as the change in reactivity per percent change in void volume. As the reactor power is raised to the point where the steam voids start to form, voids displace moderator from the coolant channels within the core. This displacement reduces the moderator-to-fuel ratio, and in an under moderated core, results in a negative reactivity addition, thereby limiting reactor power rise. The void coefficient is significant in water-moderated reactors that operate at or near saturated conditions.

SECTION FOUR: NEUTRON POISONS

In some reactors, neutron-absorbing materials called poisons are intentionally designed into the reactor for specific purposes. Some of these poisons are depleted as they absorb neutrons during reactor operation, and others remain relatively constant.

4.1. Fixed Burnable Poisons

During operation of a reactor the amount of fuel contained in the core constantly decreases. If the reactor is to operate for a long period of time, fuel in excess of that needed for exact criticality must be added when the reactor

is first loaded with fuel. The positive reactivity due to the excess fuel must be balanced with negative reactivity from neutron-absorbing material.

Moveable control rods containing neutron-absorbing material are one method used to offset the excess fuel. Control rods will be discussed in detail in another white paper. Using control rods alone to balance the excess reactivity may be undesirable or impractical for several reasons. One reason for a particular core design may be that there is physically insufficient room for the control rods and their large mechanisms.

To control large amounts of excess fuel without adding additional control rods, **burnable poisons** are loaded into the core. **Burnable poisons** are materials that have a high neutron absorption cross section that are converted into materials of relatively low absorption cross section as the result of neutron absorption. Due to the burnup of the poison material, the negative reactivity of the burnable poison decreases over the useful life of the fuel. Ideally, these poisons should decrease their negative reactivity at the same rate the fuel's excess positive reactivity is depleted.

Fixed burnable poisons are generally used in the form of compounds of **boron** or **gadolinium** that are shaped into separate lattice pins or plates, or introduced as additives to the fuel. Since they can usually be distributed more uniformly than control rods, these poisons are less disruptive to the core power distribution.

4.2. Soluble Poisons

Soluble poisons, also called **chemical shim,** produce a spatially uniform neutron absorption when dissolved in the water coolant. The most common soluble poison in **commercial pressurized water reactors** (**PWR**) is boric acid, which is often referred to as "soluble boron," or simply "solbor." The boric acid in the coolant decreases the thermal utilization factor, causing a decrease in reactivity. By varying the concentration of boric acid in the coolant (a process referred to as boration and dilution), the reactivity of the core can be easily varied.

If the boron concentration is increased, the coolant/moderator absorbs more neutrons, adding negative reactivity. The changing of boron concentration in a **PWR** is a slow process and is used primarily to compensate for fuel burnout or poison buildup. The variation in boron concentration allows control rod use to be minimized, which results in a flatter flux profile over the core than can be produced by rod insertion. The flatter flux profile is due to the fact that there are no regions of depressed flux like those that would be produced in the vicinity of inserted control rods.
Some reactors include emergency shutdown systems that inject solutions containing neutron poisons into the system that circulates reactor coolant. Various solutions, including sodium polyborate and gadolinium nitrate, are used.

Fixed burnable poisons possess some advantages over chemical shim. Fixed burnable poisons may be discretely loaded in specific locations in order to shape or control flux profiles in the core. Also, fixed burnable poisons do not make the moderator temperature reactivity coefficient less negative as chemical shim does. With chemical shim, as temperature rises and the moderator expands, some moderator is pushed out of the active core area. Boron is also moved out, and this has a positive effect on reactivity. This property of chemical shim limits the allowable boron concentration because any greater concentration makes the moderator temperature coefficient of reactivity positive.

4.3. Non-Burnable Poisons

A **non-burnable poison** is one that maintains a constant negative reactivity worth over the life of the core. While no neutron poison is strictly non-burnable, certain materials can be treated as non-burnable poisons under certain conditions. One example is **hafnium**. The removal (by absorption of neutrons) of one isotope of hafnium leads to the production of another neutron absorber, and continues through a chain of five absorbers. This absorption chain results in a long-lived burnable poison which approximates non-burnable characteristics.

Absorbers with low neutron absorption cross sections can also be treated as non-burnable under most conditions. It is possible to make the reactivity of a poison material that is usually a burnable poison more uniform over core life through the use of self-shielding. In self-shielding, the poison material is thick enough that only the outer layer of the poison is exposed to the neutron flux. The absorptions that take place in the outer layers reduce the number of neutrons that penetrate to the inner material. As the outer layers of poison absorb neutrons and are converted to non-poison materials, the inner layers begin absorbing more neutrons, and the negative reactivity of the poison is fairly uniform. The normal use of fixed non-burnable poisons is in power shaping, or to prevent excessive flux and power peaking near moderator regions of the reactor.

SECTION FIVE: XENON

Xenon-135 (**Xe^{135}**) has a tremendous impact on the operation of a nuclear reactor. It is important to understand the mechanisms that produce and remove xenon from the reactor to predict how the reactor will respond following changes in power level.

5.1. Fission Product Poisons

Fission fragments generated at the time of fission decay to produce a variety of fission products. Fission products are of concern in reactors primarily because they become parasitic absorbers of neutrons and result in long term sources of heat. Although several fission products have significant neutron absorption cross sections, **Xenon-135** (**Xe^{135}**) and **Samarium-149** (**Sm^{149}**)

have the most substantial impact on reactor design and operation. Because these two fission product poisons remove neutrons from the reactor, they will have an impact on the thermal utilization factor and thus $\mathbf{k_{eff}}$ and reactivity.

5.2. Production and Removal of Xenon-135 (Xe^{135})

Xenon-135 (Xe^{135}) has a 2.6 x 10^6 barns neutron absorption cross section. It is produced directly by some fissions, but is more commonly a product of the **tellurium-135** (Te^{135}) decay chain shown below:

$$_{52}Te^{135} \xrightarrow{\beta^-} {}_{53}I^{135} \xrightarrow{\beta^-} {}_{54}Xe^{135} \xrightarrow{\beta^-} {}_{55}Ce^{135} \xrightarrow{\beta^-} {}_{56}Ba^{135}$$

19.0 seconds | 6.57 Hrs. | 9.10 Hrs. | 2.3 x 10^6 years

The fission yield (γ) for **Xenon-135** (Xe^{135}) is about 0.3%, while γ for **Tellurium-135** (Te^{135}) is about 6%. The half-life for tellurium-135 (Te^{135}) is so short compared to the other half-lives that it can be assumed that iodine-135 (I^{135}) is produced directly from fission. Iodine-135(I^{135}) is not a strong neutron absorber, but decays to form the neutron poison xenon-135 (Xe^{135}). Ninety-five percent of all the xenon-135 (Xe^{135}) produced comes from the decay of iodine-135(I^{135}). Therefore, the half-life of iodine-135(I^{135}) plays an important role in the amount of xenon-135 (Xe^{135}). present.

The rate of change of iodine concentration is equal to the rate of production minus the rate of removal. This can be expressed in the equation below.

rate of iodine concentration = yield from fission - decay rate - burnup rate

or

$$\frac{dN_I}{dt} = \gamma_I \Sigma_f^{fuel} \varphi - \lambda_I N_I - \sigma_a^I N_I \varphi$$

where:

N_I = I^{135} concentration
γ_I = fission yield of I^{135}
Σ_f^{fuel} = macroscopic fission cross section fuel
φ = thermal neutron flux
λ_I = decay constant for I^{135}
σ_a^I = microscopic absorption cross section I^{135}

Since the σ_a^I (decay constant for Iodine) is very small, the burn up rate term may be ignored, and the expression for the rate of change of iodine concentration is modified as shown below:

$$\frac{dN_I}{dt} = \gamma_I \Sigma_f^{fuel} \varphi - \lambda_I N_I$$

When the rate of production of iodine equals the rate of removal of iodine, equilibrium exists. The iodine concentration then remains constant and is

designated $N^I(eq)$. The following equation for the equilibrium concentration of iodine can be determined from the preceding equation by setting the two terms equal to each other and solving for $N^I(eq)$.

$$N_I = \frac{\gamma_I \Sigma_f^{fuel} \varphi}{\lambda_I}$$

Since the equilibrium iodine concentration is proportional to the fission reaction rate, it is also proportional to reactor power level.

The rate of change of the xenon concentration is equal to the rate of production minus the rate of removal. Recall that 5% of xenon comes directly from fission and 95% comes from the decay of iodine. The rate of change of xenon concentration is expressed by the following equations:

rate of change of Xe^{135} present = (**Xe^{135} yield from fission**) + (**I^{135} decay**) - (**Xe^{135} decay**) + (**Xe^{135} Burn-up**)

$$\frac{dN_{Xe}}{dt} = \gamma_{Xe} \Sigma_f^{fuel} \varphi + \lambda_I N_I - \lambda_{Xe} N_{Xe} - \sigma_a^{Xe} N_{Xe} \varphi$$

where:

N_{Xe} = Xe^{135} concentration
γ_{Xe} = fission yield of Xe^{135}
Σ_f^{fuel} = macroscopic fission cross section fuel
φ = thermal neutron flux
λ_I = decay constant for I^{135}
N_I = I^{135} concentration
λ_{Xe} = decay constant for Xe^{135}
σ_a^{Xe} = microscopic absorption cross section Xe^{135}

The xenon burn-up term above refers to neutron absorption by **Xenon-135** (Xe^{135}) by the following reaction:

$$_{54}Xe^{135} + {}_0n^1 \rightarrow {}_{54}Xe^{136} + \gamma$$

Xenon-136 (Xe^{136}) is not a significant neutron absorber; therefore, the neutron absorption by **Xenon-135** (Xe^{135}) constitutes removal of poison from the reactor. The burnup rate of **Xenon-135** (Xe^{135}) is dependent upon the neutron flux and the **Xenon-135** (Xe^{135}) concentration.

The equilibrium concentration of **Xenon-135** (Xe^{135}) is designated $N_{Xe}(eq)$, and is represented as shown below:

$$N_{Xe}(eq) = \frac{\gamma_{Xe} \Sigma_f^{fuel} \varphi + \lambda_I N_I}{\lambda_{Xe} + \sigma_a^{Xe} \varphi}$$

where:

N_{Xe} = Xe^{135} equilibrium concentration

γ_{Xe} = fission yield of **Xe^{135}**
Σ_f^{fuel} = macroscopic fission cross section fuel
φ = thermal neutron flux
λ_I = decay constant for **I^{135}**
N_I = **I^{135}** concentration
λ_{Xe} = decay constant for **Xe^{135}**
σ_a^{Xe} = microscopic absorption cross section **Xe^{135}**

For **Xenon-135** (**Xe^{135}**) to be in equilibrium, **Iodine-135** (**I^{135}**) must also be in equilibrium. Substituting the expression for equilibrium **Iodine-135** (**I^{135}**) concentration into the equation for equilibrium xenon results in the following:

$$N_{Xe}(eq) = \frac{(\gamma_{Xe} + \gamma_I)\Sigma_f^{fuel}\varphi + \lambda_I N_I}{\lambda_{Xe} + \sigma_a^{Xe}\varphi}$$

where:

N_{Xe} = **Xe^{135}** equilibrium concentration
γ_{Xe} = fission yield of **Xe^{135}**
γ_I = fission yield of **I^{135}**
Σ_f^{fuel} = macroscopic fission cross section fuel
φ = thermal neutron flux
λ_I = decay constant for **I^{135}**
N_I = **I^{135}** concentration
λ_{Xe} = decay constant for **Xe^{135}**
σ_a^{Xe} = microscopic absorption cross section **Xe^{135}**

From this equation it can be seen that the equilibrium value for **Xenon-135** (**Xe^{135}**) increases as power increases, because the numerator is proportional to the fission reaction rate.

Thermal flux is also in the denominator; therefore, as the thermal flux exceeds 10^{12} neutrons/cm^2-sec, the term begins to dominate, and at approximately 10^{15} neutrons/cm^2-sec, the **Xenon-135** (**Xe^{135}**) concentration approaches a limiting value.

The higher the power level, or flux, the higher the equilibrium **Xenon-135** (**Xe^{135}**) concentration, but equilibrium **Xenon-135** (**Xe^{135}**) is not directly proportional to power level. For example, equilibrium **Xenon-135** (**Xe^{135}**) at 25% power is more than half the value for equilibrium **Xenon-135** (**Xe^{135}**) at 100% power for many reactors. Because the **Xenon-135** (**Xe^{135}**) concentration directly affects the reactivity level in the reactor core, the negative reactivity due to the xenon concentrations for different power levels or conditions are frequently plotted instead of the xenon concentration.

5.3 Xenon-135 Response to Reactor Shutdown

When a reactor is shutdown, the neutron flux is reduced essentially to zero. Therefore, after shutdown, **Xenon-135** (**Xe^{135}**) is no longer produced by

fission and is no longer removed by burnup. The only remaining production mechanism is the decay of the , **Iodine-135** (**I^{135}**) which was in the core at the time of shutdown. The only removal mechanism for **Xenon-135** (**Xe^{135}**) is decay.

$$\frac{d N_{Xe}}{dt} = \lambda_I N_I - \lambda_{Xe} N_{Xe}$$

where:

N_{Xe} = **Xe^{135}** concentration
λ_I = decay constant for **I^{135}**
N_I = **I^{135}** concentration
λ_{Xe} = decay constant for **Xe^{135}**

Because the decay rate of iodine-135 is faster than the decay rate of xenon-135, the xenon concentration builds to a peak. The peak is reached when the product of the terms lINI is equal to lXeNXe (in about 10 to 11 hours). Subsequently, the production from iodine decay is less than the removal of xenon by decay, and the concentration of **Xenon-135** (**Xe^{135}**) decreases. The greater the flux level prior to shutdown, the greater the concentration of **Iodine-135** (**I^{135}**) at shutdown; therefore, the greater the peak in **Xenon-135** (**Xe^{135}**) concentration after shutdown.

Negative **Xenon-135** (**Xe^{135}**) reactivity, also called **Xenon Poisoning,** may provide sufficient negative reactivity to make the reactor inoperable because there is insufficient positive reactivity available from control rod removal or chemical shim dilution (if used) to counteract it. The inability of the reactor to be started due to the effects of **Xenon-135** (**Xe^{135}**) is sometimes referred to as a **xenon precluded startup**.

The period of time where the reactor is unable to "override" the effects of **xenon** is called **xenon dead time**. Because the amount of excess core reactivity available to override the negative reactivity of the **Xenon-135** (**Xe^{135}**) is usually less than 10%**Δk/k,** thermal power reactors are normally limited to flux levels of about 5 x 10^{13} neutrons/cm^2 -sec so that timely restart can be ensured after shutdown. For reactors with very low thermal flux levels (~5 x 10^{12} neutrons/cm^2-sec or less), most **Xenon-135** (**Xe^{135}**) is removed by decay as opposed to neutron absorption. For these cases, reactor shutdown does not cause any **Xenon-135** (**Xe^{135}**) peaking effect.

Following the peak in **Xenon-135** (**Xe^{135}**) concentration about 10 hours after shutdown, the **Xenon-135** (**Xe^{135}**) concentration will decrease at a rate controlled by the decay of **Iodine-135** (**I^{135}**) into **Xenon-135** (**Xe^{135}**) and the decay rate of **Xenon-135** (**Xe^{135}**). For some reactors, the **Xenon-135** (**Xe^{135}**) concentration about 20 hours after shutdown from full power will be the same as the equilibrium **Xenon-135** (**Xe^{135}**) concentration at full power. About 3 days after shutdown, the **Xenon-135** (**Xe^{135}**) concentration will have decreased to a small percentage of its pre-shutdown level, and the reactor

can be assumed to be **Xenon-135** (**Xe^{135}**) free without a significant error introduced into reactivity calculations.

5.4 Xenon-135 (Xe^{135}) Oscillations

Large thermal reactors with little flux coupling between regions may experience spatial power oscillations because of the non-uniform presence of **Xenon-135** (**Xe^{135}**). The mechanism is described in the following four steps:

1. An initial lack of symmetry in the core power distribution (for example, individual control rod movement or misalignment) causes an imbalance in fission rates within the reactor core, and therefore, in the iodine-135 buildup and the **Xenon-135** (**Xe^{135}**) xenon-135 absorption.

2. In the high-flux region, **Xenon-135** (**Xe^{135}**) xenon-135 burnout allows the flux to increase further, while in the low-flux region, the increase in **Xenon-135** (**Xe^{135}**) causes a further reduction in flux. The iodine concentration increases where the flux is high and decreases where the flux is low.

3. As soon as the **Iodine-135** (**I^{135}**) levels build up sufficiently, decay to **Xenon** reverses the initial situation. Flux decreases in this area, and the former low-flux region increases in power.

4. Repetition of these patterns can lead to **Xenon** oscillations moving about the core with periods on the order of about 15 hours. With little change in overall power level, these oscillations can change the local power levels by a factor of three or more. In a reactor system with strongly negative temperature coefficients, the **Xenon-135** (**Xe^{135}**) oscillations are damped quite readily. This is one reason for designing reactors to have negative moderator-temperature coefficients.

5.5 Xenon-135 (Xe^{135}) Response to Reactor Power Changes

During periods of steady state operation, at a constant neutron flux level, the **Xenon-135** (**Xe^{135}**) concentration builds up to its equilibrium value for that reactor power in about 40 to 50 hours. Most **Xenon-135** (**Xe^{135}**) transients occurs as a result of a change in reactor power level. At time zero, reactor power is raised from 50% power to 100% power. When the reactor power is increased, **Xenon-135** (**Xe^{135}**) xenon concentration initially decreases because the burnup is increased at the new higher power level.

Because 95% of the **Xenon-135** (**Xe^{135}**) production is from **Iodine-135** (**I^{135}**) decay, which has a 6 to 7 hour half-life, the production of **Xenon-135** (**Xe^{135}**) remains constant for several hours. After a few hours (roughly 4 to 6 hours depending on power levels) the rate of production of **Xenon-135** (**Xe^{135}**) from iodine and fission equals the rate of removal of **Xenon-135** (**Xe^{135}**) by burnup and decay. At this point, the **Xenon-135** (**Xe^{135}**) concentration reaches a minimum.

The **Xenon-135** (**Xe^{135}**) xenon concentration then increases to the new equilibrium level for the new power level in roughly 40 to 50 hours. It should

be noted that the magnitude and the rate of change of **Xenon-135** (Xe^{135}) concentration during the initial 4 to 6 hours following the power change is dependent upon the initial power level and on the amount of change in power level. The **Xenon concentration change** is greater for a larger change in power level.

When reactor power is decreased from 100% to 50% power (t = 55 hours), the process is reversed. There is an immediate decrease in **Xenon** burnup, which results in an increase in **Xenon-135** (Xe^{135}) concentration. The **Iodine-135** (I^{135}) concentration is still at the higher equilibrium level for 100% power and is therefore still producing **Xenon-135** (Xe^{135}) at the higher rate. The **Xenon-135** (Xe^{135}) concentration continues to rise until the rate of production of **Xenon-135** (Xe^{135}) becomes equal to the rate of removal (roughly 7 to 8 hours after the initial reduction in power level). The **Xenon-135** (Xe^{135}) concentration then gradually decreases to the new equilibrium level in about 50 to 60 hours.

The magnitude of the **Xenon** peak is greatest if the initial power level is very high. Maximum peak **Xenon** occurs when a reactor that is operating at 100% equilibrium **Xenon** concentration is suddenly shut down. The most rapid possible burnout of **Xenon** occurs when a reactor is started up and operated at full power while this maximum peak xenon condition exists.

SECTION SIX: SAMARIUM AND OTHER FISSION PRODUCT POISONS

6.1 Production and Removal of Samarium-149 (Sm^{149})

The fission product poison that has the most significant effect on reactor operations other than **Xenon-135** (Xe^{135}) is **Samarium-149** (Sm^{149}). **Samarium-149** (Sm^{149}) behaves significantly different from **Xenon-135** (Xe^{135}) due to its different nuclear properties.

Samarium-149 (Sm^{149}) is the second most important fission-product poison because of its high thermal neutron absorption cross section of 4.1×10^4 barns. **Samarium-149** (Sm^{149}) is produced from the decay of the **Neodymium-149** (Nd^{149}) fission fragment. For the purpose of examining the behavior of **Samarium-149** (Sm^{149}), the 1.73 hour half-life of **Neodymium-149** (Nd^{149}) is sufficiently shorter than the 53.1 hour value for **Promethium-149** (Pm^{149}) that the **Promethium-149** (Pm^{149}) may be considered as if it were formed directly from fission. This assumption, and neglecting the small amount of **Promethium-149** (Pm^{149}) burnup, allows the situation to be described as follows:

Rate of change of Pm^{149} = yield from fission - decay Pm^{149} concentration

therefore:

$$\frac{dN_{Pm}}{dt} = \gamma_{Pm} \Sigma_f^{fuel} \varphi + \lambda_{Pm} N_{Pm}$$

where:

N_{Pm} = Pm^{149} equilibrium concentration

γ_{Pm} = fission yield of **Pm^{149}**
Σ_f^{fuel} = macroscopic fission cross section fuel
φ = thermal neutron flux
λ_{Pm} = decay constant for **Pm^{149}**

Solving for the equilibrium value of **Promethium-149** (**Pm^{149}**) gives the following:

$$N_{Pm}(eq.) = \frac{\gamma_{Pm} \Sigma_f^{fuel} \varphi}{\lambda_{Pm}}$$

where:

$N_{Pm}(eq.)$ = **Pm^{149}** equilibrium concentration
γ_{Pm} = fission yield of **Pm^{149}**
Σ_f^{fuel} = macroscopic fission cross section fuel
φ = thermal neutron flux
λ_{Pm} = decay constant for **Pm^{149}**

The rate of **Samarium-149** (**Sm^{149}**) formation is described as follows:

Rate of change of Sm^{149} = yield from fission + Pm^{149} decay - Sm^{149} burnup

therefore:

$$\frac{d N_{Sm}}{dt} = \gamma_{Sm} \Sigma_f^{fuel} \varphi + \lambda_{Pm} N_{Pm} - \sigma_a^{Sm} N_{Sm} \varphi$$

where:

N_{Sm} = **Sm^{149}** concentration
γ_{Sm} = fission yield of **Sm^{149}**
Σ_f^{fuel} = macroscopic fission cross section fuel
φ = thermal neutron flux
λ_{Pm} = decay constant for **Pm^{149}**
N_{Pm} = **Pm^{149}** concentration
λ_{Xe} = decay constant for **Xe^{135}**
σ_a^{Sm} = microscopic absorption cross section **Sm^{149}**

The fission yield of **Samarium-149** (**Sm^{149}**), however, is nearly zero; therefore, the equation becomes the following:

$$\frac{d N_{Sm}}{dt} = \lambda_{Pm} N_{Pm} - \sigma_a^{Sm} N_{Sm} \varphi$$

Solving this equation for the equilibrium concentration of **Samarium-149** (**Sm^{149}**) and substituting

$$N_{Pm}(eq.) = \frac{\gamma_{Pm} \Sigma_f^{fuel} \varphi}{\lambda_{Pm}}$$

for the equilibrium concentration of **Promethium-149** (**Pm^{149}**), yields:

$$N_{Sm}(eq.) = \frac{\gamma_{Pm} \Sigma_f^{fuel} \varphi}{\lambda_{Sm}}$$

This expression for equilibrium **Samarium-149** (**Sm^{149}**) concentration during reactor operation illustrates that equilibrium **Samarium-149** (**Sm^{149}**), concentration is independent of neutron flux and power level. The **Samarium** concentration will undergo a transient following a power level change, but it will return to its original value.

6.2 Samarium-149 Response to Reactor Shutdown

Since the neutron flux drops to essentially zero after reactor shutdown, the rate of samarium-149 production becomes the following.

$$\frac{d N_{Sm}}{dt} = \lambda_{Pm} N_{Pm}$$

where:

N_{Sm} = **Sm^{149}** concentration
λ_{Pm} = decay constant for **Pm^{149}**
N_{Pm} = **Pm^{149}** concentration

Because **Samarium-149** (**Sm^{149}**) is not radioactive and is not removed by decay, it presents problems somewhat different from those encountered with **Xenon-135** (**Xe^{135}**). The equilibrium concentration and the poisoning effect build to an equilibrium value during reactor operation. This equilibrium is reached in approximately 20 days (500 hours), and since **Samarium-149** (**Sm^{149}**) is stable, the concentration remains essentially constant during reactor operation.

When the reactor is shutdown, the **Samarium-149** (**Sm^{149}**) concentration builds up as a result of the decay of the accumulated **Promethium-149** (**Pm^{149}**). The buildup of **Samarium-149** (**Sm^{149}**) after shutdown depends upon the power level before shutdown. **Samarium-149** (**Sm^{149}**) does not peak as **Xenon-135** (**Xe^{135}**) does, but increases slowly to a maximum value.

After shutdown, if the reactor is then operated at power, **Samarium-149** (**Sm^{149}**) is burned up and its concentration returns to the equilibrium value. **Samarium-149** (**Sm^{149}**) poisoning is minor when compared to **Xenon-135** (**Xe^{135}**) poisoning. Although **Samarium-149** (**Sm^{149}**) has a constant poisoning effect during long-term sustained operation, its behavior during initial startup and during post-shutdown and restart periods requires special considerations in reactor design.

The **Xenon-135** (**Xe^{135}**) and **Samarium-149** (**Sm^{149}**) mechanisms are dependent on their very large thermal neutron cross sections and only affect thermal reactor systems. In fast reactors, neither these nor any other fission products have a major poisoning influence.

6.3 Other Neutron Poisons

There are numerous other fission products that, as a result of their concentration and thermal neutron absorption cross section, have a poisoning effect on reactor operation. Individually, they are of little consequence, but "lumped" together they have a significant impact. These are often characterized as **lumped fission product poisons** and accumulate at an average rate of 50 barns per fission event in the reactor. In addition to fission product poisons, other materials in the reactor decay to materials that act as neutron poisons. An example of this is the decay of tritium to helium-3. Since tritium has a half-life of 12.3 years, normally this decay does not significantly affect reactor operations because the rate of decay of tritium is so slow. However, if tritium is produced in a reactor and then allowed to remain in the reactor during a prolonged shutdown of several months, a sufficient amount of tritium may decay to helium-3 to add a significant amount of negative reactivity.

SECTION SEVEN: CONTROL RODS

Most reactors contain control rods made of neutron absorbing materials that are used to adjust the reactivity of the core. Control rods can be designed and used for coarse control, fine control, or fast shutdowns.

7.1. Selection of Control Rod Materials

Rods of neutron-absorbing material are installed in most reactors to provide precise, adjustable control of reactivity. These rods are able to be moved into or out of the reactor core and typically contain elements such as silver, indium, cadmium, boron, or hafnium. The material used for the control rods varies depending on reactor design. Generally, the material selected should have a good absorption cross section for neutrons and have a long lifetime as an absorber (not burn out rapidly).

The ability of a control rod to absorb neutrons can be adjusted during manufacture. A control rod that is referred to as a "black" absorber absorbs essentially all incident neutrons. A "grey" absorber absorbs only a part of them. While it takes more grey rods than black rods for a given reactivity effect, the grey rods are often preferred because they cause smaller depressions in the neutron flux and power in the vicinity of the rod. This leads to a flatter neutron flux profile and more even power distribution in the core.

If grey rods are desired, the amount of material with a high absorption cross section that is loaded in the rod is limited. Material with a very high absorption cross section may not be desired for use in a control rod, because it will burn out rapidly due to its high absorption cross section. The same amount of reactivity worth can be achieved by manufacturing the control rod from material with a slightly lower cross section and by loading more of the material. This also results in a rod that does not burn out as rapidly.

Another factor in control rod material selection is that materials that resonantly absorb neutrons are often preferred to those that merely have high thermal neutron absorption cross sections. Resonance neutron absorbers absorb neutrons in the epithermal energy range. The path length traveled by the epithermal neutrons in a reactor is greater than the path length traveled by thermal neutrons. Therefore, a resonance absorber absorbs neutrons that have their last collision farther (on the average) from the control rod than a thermal absorber. This has the effect of making the area of influence around a resonance absorber larger than around a thermal absorber and is useful in maintaining a flatter flux profile.

7.2. Types of Control Rods

There are several ways to classify the types of control rods. One classification method is by the purpose of the control rods. Three purposes of control rods are listed below.

A. **Shim rods**: used for coarse control and/or to remove reactivity in relatively large amounts.

B. **Regulating rods:** used for fine adjustments and to maintain desired power or temperature.

C. **Safety rods:** provide a means for very fast shutdown in the event of an unsafe condition. Addition of a large amount of negative reactivity by rapidly inserting the safety rods is referred to as a "scram" or "trip."

Not all reactors have different control rods to serve the purposes mentioned above. Depending upon the type of reactor and the controls necessary, it is possible to use dual-purpose or even triple-purpose rods. For example, consider a set of control rods that can insert enough reactivity to be used as shim rods. If the same rods can be operated at slow speeds, they will function as regulating rods. Additionally, these same rods can be designed for rapid insertion, or scram. These rods serve a triple function yet meet other specifications such as precise control, range of control, and efficiency.

7.3. Control Rod Effectiveness

The effectiveness of a control rod depends largely upon the value of the ratio of the neutron flux at the location of the rod to the average neutron flux in the reactor. The control rod has maximum effect (inserts the most negative reactivity) if it is placed in the reactor where the flux is maximum. If a reactor has only one control rod, the rod should be placed in the center of the reactor core.

If additional rods are added to this simple reactor, the most effective location is where the flux is maximum. Numerous control rods are required for a reactor that has a large amount of excess reactivity (that amount of reactivity in excess of that needed to be critical). The exact amount of reactivity that

each control rod inserts depends upon the reactor design. The change in reactivity caused by control rod motion is referred to as control rod worth.

7.4. Integral and Differential Control Rod Worth

The exact effect of control rods on reactivity can be determined experimentally. For example, a control rod can be withdrawn in small increments, such as 0.5 inch, and the change in reactivity can be determined following each increment of withdrawal. By plotting the resulting reactivity versus the rod position, a graph is obtained. The graph depicts integral control rod worth over the full range of withdrawal. The **integral control rod worth** is the total reactivity worth of the rod at that particular degree of withdrawal and is usually defined to be the greatest when the rod is fully withdrawn.

The slope of the curve ($\Delta\rho/\Delta x$), and therefore the amount of reactivity inserted per unit of withdrawal, is greatest when the control rod is midway out of the core. This occurs because the area of greatest neutron flux is near the center of the core. If the slope of the curve for integral rod worth is taken, the result is a value for rate of change of control rod worth as a function of control rod position.

At the bottom of the core, where there are few neutrons, rod movement has little effect so the change in rod worth per inch varies little. As the rod approaches the center of the core its effect becomes greater, and the change in rod worth per inch is greater. At the center of the core the differential rod worth is greatest and varies little with rod motion. From the center of the core to the top, the rod worth per inch is basically the inverse of the rod worth per inch from the center to the bottom.

Differential control rod worth is the reactivity change per unit movement of a rod and is normally expressed as **ρ/inch**, **$\Delta k/k$ per inch**, or **pcm/inch**. The integral rod worth at a given withdrawal is merely the summation of all the differential rod worths up to that point of withdrawal. It is also the area under the differential rod worth curve at any given withdrawal position.

7.5. Rod Control Mechanisms

The control rod insertion rates on a scram are designed to be sufficient to protect the reactor against damage in all transients that are expected to occur during the life of the reactor. During normal rod motion, the control rods must be able to move rapidly enough to compensate for the most rapid rate at which positive reactivity is expected to build within the reactor in order to provide positive control. The transient that is normally considered when setting this minimum rod speed is the burnout of maximum peak xenon while at full power. Xenon burnout is usually the most rapid, non-accident transient expected. The maximum rod speed is normally limited in order to reduce the severity of an accident involving the continuous withdrawal of control rods.

END OF CHAPTER THREE: INTRODUCTION TO REACTOR PHYSICS

CHAPTER FOUR:INTRODUCTION TO REACTOR OPERATIONS

SECTION ONE: SUB-CRITICAL MULTIPLICATION

1.1 Sub-critical Multiplication Factor

When a reactor is in a shutdown condition, neutrons are still present to interact with the fuel. These source neutrons are produced by a variety of methods that were discussed in the white paper on Neutron Physics. If neutrons and fissionable material are present in the reactor, fission will take place. Therefore, a reactor will always be producing a small number of fissions even when it is shutdown.

Consider a reactor in which $\mathbf{k_{eff}}$ is 0.6. If 100 neutrons are suddenly introduced into the reactor, these 100 neutrons that start the current generation will produce 60 neutrons (100 x 0.6) from fission to start the next generation. The 60 neutrons that start the second generation will produce 36 neutrons (60 x 0.6) to start the third generation.

Because the reactor is sub-critical, neutrons introduced in the reactor will have a decreasing effect on each subsequent generation. The addition of source neutrons to the reactor containing fissionable material has the effect of maintaining a much higher stable neutron level due to the fissions occurring than the neutron level that would result from the source neutrons alone. A neutron source strength of 100 neutrons per generation will result in 250 neutrons per generation being produced from a combination of sources and fission in a shutdown reactor with a $\mathbf{k_{eff}}$ of 0.6. If the value of $\mathbf{k_{ef}}$ were higher, the source neutrons would produce a greater number of fission neutrons and their effects would be felt for a larger number of subsequent generations after their addition to the reactor.

The effect of fissions in the fuel increasing the effective source strength of a reactor with a $\mathbf{k_{eff}}$ of less than one is **sub-critical multiplication**. For a given value of $\mathbf{k_{eff}}$ there exists a **sub-critical multiplication factor** (**M**) that relates the source level to the steady-state neutron level of the core.

If the value of $\mathbf{k_{eff}}$ is known, the amount that the neutron source strength will be multiplied (M) can easily be determined by the Equation below:

$$\mathbf{M = \frac{1}{1 - k_{eff}}}$$

where:

M = sub-critical multiplication factor
$\mathbf{k_{eff}}$ = effective multiplication factor

The equation above illustrates that the sub-critical multiplication factor will increase as positive reactivity is added to a shutdown reactor, increasing the value of $\mathbf{k_{eff}}$. If the source strength of this reactor were 1000 neutrons/sec, the

neutron level would increase from 2500 neutrons/second at a k_{eff} keff of 0.6 to a neutron level of 71,400 neutrons/sec at a keff of 0.986.

1.2 Effect of Reactivity Changes on Sub-critical Multiplication

In a sub-critical reactor, the neutron level is related to the source strength by the following equation:

$$N = S \times M$$

where:

N = neutron level
S = neutron source strength
M = sub-critical multiplication factor

If the term **M** in the previous equation is replaced by the expression:

$$M = \frac{1}{1 - k_{eff}}$$

the following expression results:

$$N = S \times \frac{1}{(1 - k_{eff})}$$

To this point it has been necessary to know the neutron source strength of the reactor in order to use the concept of **sub-critical multiplication**. In most reactors the actual strength of the neutron sources is difficult, if not impossible, to determine. Even though the actual source strength may not be known, it is still possible to relate the change in reactivity to a change in neutron level.

Consider a reactor at two different times when k_{eff} is two different values, k_{eff1} and k_{eff2}. The neutron level at each time can be determined based on the neutron source strength and the **sub-critical multiplication factor** using our previous equation:

$$N_1 = \frac{S \times 1}{(1 - k_{eff1})} \quad \text{and}$$

$$N_2 = \frac{S \times 1}{(1 - k_{eff2})}$$

The equation for N_1 can be divided by N_2 and

$$\frac{N_1}{N_2} = \frac{(1 - k_{eff2})}{(1 - k_{eff1})}$$

Because the source strength appears in both the numerator and denominator, it cancels out of the equation. Therefore, the neutron level at any time can be determined based on the neutron level present at any other time provided the values of k_{eff} or reactivity for both times are known.

The neutron level in a shutdown reactor is typically monitored using instruments that measure the neutron leakage out of the reactor. The neutron

leakage is proportional to the neutron level in the reactor. Typical units for displaying the instrument reading are **counts per second** (**cps**). Because the instrument count rate is proportional to the neutron level, the above equation can be restated as shown below:

$$\frac{CR_1}{CR_2} = \frac{(1 - k_{eff2})}{(1 - k_{eff1})}$$

where:

CR_1 = count rate at time 1
CR_2 = count rate at time 2
k_{eff1} = k_{eff} at time 1
k_{eff2} = k_{eff} at time 2

This equation is very useful during the shutdown operation of a reactor. Before adding positive reactivity to a reactor, it is possible to predict the effect the reactivity addition will have on the neutron level.

1.3 Use of 1/M Plots

Because the sub-critical multiplication factor is related to the value of k_{eff}, it is possible to monitor the approach to criticality through the use of the sub-critical multiplication factor. As positive reactivity is added to a sub-critical reactor, k_{eff} will get nearer to one. As k_{eff} gets nearer to one, the sub-critical multiplication factor (**M**) gets larger.

The closer the reactor is to criticality, the faster **M** will increase for equal step insertions of positive reactivity. When the reactor becomes critical, **M** will be infinitely large. For this reason, monitoring and plotting **M** during an approach to criticality is impractical because there is no value of **M** at which the reactor clearly becomes critical. Instead of plotting **M** directly, its inverse (**1/M**) is plotted on a graph of **1/M** versus rod height:

$$M = \frac{1}{1 - k_{eff}}$$

$$\frac{1}{M} = 1 - k_{eff}$$

As control rods are withdrawn and k_{eff} approaches one and **M** approaches infinity, **1/M** approaches zero. For a critical reactor, 1/M is equal to zero. A true 1/M plot requires knowledge of the neutron source strength. Because the actual source strength is usually unknown, a reference count rate is substituted, and the calculation of the factor **1/M** is through the use of the following equation:

$$\frac{1}{M} = \frac{CR_0}{CR}$$

where: **1/M** = inverse multiplication factor
CR_0 = reference count rate
CR = current count rate

In practice, the reference count rate used is the count rate prior to the beginning of the reactivity change. The startup procedures for many reactors include instructions to insert positive reactivity in incremental steps with delays between the reactivity insertions to allow time for sub-critical multiplication to increase the steady-state neutron population to a new, higher level and allow more accurate plotting of **1/M.**

The neutron population will typically reach its new steady-state value within 1-2 minutes, but the closer the reactor is to criticality, the longer the time will be to stabilize the neutron population.

SECTION TWO: REACTOR KINETICS

The response of neutron flux and reactor power to changes in reactivity is much different in a critical reactor than in a sub-critical reactor. The reliance of the chain reaction on delayed neutrons makes the rate of change of reactor power controllable.

2.1 Reactor Period ( )

The **reactor period** is defined as the time required for reactor power to change by a factor of "e," where "e" is the base of the natural logarithm and is equal to about 2.718. The reactor period is usually expressed in units of seconds. From the definition of reactor period, it is possible to develop the relationship between reactor power and reactor period that is expressed by the following equation:

$$P = P_0 \times e^{t/\tau}$$

where:

P = transient reactor power
P_0 = initial reactor power
τ = reactor period (seconds)
t = time during the reactor transient (seconds)

The smaller the value of τ, the more rapid the change in reactor power. If the reactor period is positive, reactor power is increasing. If the reactor period is negative, reactor power is decreasing.

There are numerous equations used to express reactor period, but the following equation will be useful in most situations. The first term in this equation is the prompt term and the second term is the delayed term:

$$\tau = \frac{\ell^*}{\rho} + \frac{\beta_{eff} - \rho}{\lambda_{eff}\,\rho - (d\rho/dt)}$$

where:

τ = Reactor Period

ℓ^* = prompt generation lifetime

β_{eff} = effective delayed neutron fraction

ρ = reactivity

λ_{eff} = effective delayed neutron precursor decay constant

$d\rho/dt$ = rate of change of reactivity

2.2 Effective Delayed Neutron Fraction

Recall that β, the **delayed neutron fraction**, is the fraction of all fission neutrons that are born as delayed neutrons. The value of β depends upon the actual nuclear fuel used. As discussed in our first white paper, the delayed neutron precursors for a given type of fuel are grouped on the basis of half-life.

The term $\bar{\beta}$ (pronounced beta-bar) is the **average delayed neutron fraction**. The value of $\bar{\beta}$ is the weighted average of the total delayed neutron fractions of the individual types of fuel. Each total delayed neutron fraction value for each type of fuel is weighted by the percent of total neutrons that the fuel contributes through fission. If the percentage of fissions occurring in the different types of fuel in a reactor changes over the life of the core, the average delayed neutron fraction will also change.

For a light water reactor using low enriched fuel, the average delayed neutron fraction can change from 0.0070 to 0.0055 as **Uranium-235** (**U^{235}**) is burned out and **Plutonium-239** (**Pu^{239}**) is produced from **Uranium-238** (**U^{238}**).

Delayed neutrons do not have the same properties as prompt neutrons released directly from fission. The average energy of prompt neutrons is about 2 MeV. This is much greater than the average energy of delayed neutrons (about 0.5 MeV). The fact that delayed neutrons are born at lower energies has two significant impacts on the way they proceed through the neutron life cycle.

1. Delayed neutrons have a much lower probability of causing fast fissions than prompt neutrons because their average energy is less than the minimum required for fast fission to occur.
2. Delayed neutrons have a lower probability of leaking out of the core while they are at fast energies, because they are born at lower energies and subsequently travel a shorter distance as fast neutrons.

These two considerations (lower fast fission factor and higher fast non-leakage probability for delayed neutrons) are taken into account by a term called the **importance factor** (**I**). The importance factor relates the **average delayed neutron fraction** to the **effective delayed neutron fraction.**

The **effective delayed neutron fraction** ($\bar{\beta}_{eff}$) is defined as the fraction of neutrons at thermal energies which were born delayed. The effective delayed

neutron fraction is the product of the average delayed neutron fraction and the importance factor.

$$\bar{\beta}_{eff} = \bar{\beta} \times I$$

where: $\bar{\beta}_{eff}$ = effective delayed neutron fraction
$\bar{\beta}$ = average delayed neutron fraction
I = importance factor

In a small reactor with highly enriched fuel, the increase in fast non-leakage probability will dominate the decrease in the fast fission factor, and the importance factor will be greater than one. In a large reactor with low enriched fuel, the decrease in the fast fission factor will dominate the increase in the fast non-leakage probability and the importance factor will be less than one (about 0.97 for a commercial **PWR**).

2.3 Effective Delayed Neutron Precursor Decay Constant

Another new term has been introduced in the **reactor period** (τ) equation. That term is λ_{eff} (pronounced lambda effective), the **effective delayed neutron precursor decay constant.** The decay rate for a given delayed neutron precursor can be expressed as the product of precursor concentration and the decay constant (λ) of that precursor. The decay constant of a precursor is simply the fraction of an initial number of the precursor atoms that decays in a given unit time.

A decay constant of 0.1 sec^{-1}, for example, implies that one-tenth, or ten percent, of a sample of precursor atoms decays within one second. The value for the effective delayed neutron precursor decay constant, λ_{eff} , varies depending upon the balance existing between the concentrations of the precursor groups and the nuclide(s) being used as the fuel.

If the reactor is operating at a constant power, all the precursor groups reach an equilibrium value. During an up-power transient, however, the shorter-lived precursors decaying at any given instant were born at a higher power level (or flux level) than the longer-lived precursors decaying at the same instant. There is, therefore, proportionately more of the shorter-lived and fewer of the longer-lived precursors decaying at that given instant than there are at constant power. The value of λ_{eff} is closer to that of the shorter-lived precursors.

During a down-power transient the longer-lived precursors become more significant. The longer-lived precursors decaying at a given instant were born at a higher power level (or flux level) than the shorter-lived precursors decaying at that instant. Therefore, proportionately more of the longer-lived precursors are decaying at that instant, and the value of λ_{eff} approaches the values of the longer-lived precursors.
Approximate values for λ_{eff} are 0.08 sec for steady-state operation, 0.1 sec for a power increase, and 0.05 sec^{-1} for a power decrease. The exact values will

depend upon the materials used for fuel and the value of the reactivity of the reactor core.

$$\tau = \frac{\ell^*}{\rho} + \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\,\rho - (d\rho/dt)}$$

where:

τ = Reactor Period
ℓ^* = prompt generation lifetime
$\bar{\beta}_{eff}$ = effective delayed neutron fraction
ρ = reactivity
λ_{eff} = effective delayed neutron precursor decay constant
$d\rho/dt$ = rate of change of reactivity

The first term of this equation is the prompt term and the second term is the delayed term of the Nuclear Reaction. If the positive reactivity added is less than the value of $\bar{\beta}_{eff}$, the emission of prompt fission neutrons alone is not sufficient to overcome losses to non-fission absorption and leakage. If delayed neutrons were not being produced, the neutron population would decrease as long as the reactivity of the core has a value less than the effective delayed neutron fraction. The positive reactivity insertion is followed immediately by a small immediate power increase called the *prompt jump*. This power increase occurs because the rate of production of prompt neutrons changes abruptly as the reactivity is added. Recall from an earlier module that the generation time for prompt neutrons is on the order of 10-13 seconds. The effect can be seen in Figure 2.

After the prompt jump, the rate of change of power cannot increase any more rapidly than the built-in time delay the precursor half-lives allow. Therefore, the power rise is controllable, and the reactor can be operated safely.

Conversely, in the case where negative reactivity is added to the core there will be a prompt drop in reactor power. The **prompt drop** is the small immediate decrease in reactor power caused by the negative reactivity addition. After the prompt drop, the rate of change of power slows and approaches the rate determined by the delayed term of the equation above.

2.4 Prompt Criticality

It can be readily seen from the equation above that if the amount of positive reactivity added equals the value of $\bar{\beta}_{eff}$, the reactor period equation becomes the following:

$$\tau = \frac{\ell^*}{\rho}$$

where: τ = Reactor Period
ℓ^* = prompt generation lifetime

In this case, the production of prompt neutrons alone is enough to balance neutron losses and increase the neutron population. The condition where the reactor is critical on prompt neutrons, and the neutron population increases as rapidly as the prompt neutron generation lifetime allows is known as **prompt critical**.

The **prompt critical** condition does not signal a dramatic change in neutron behavior. The reactor period changes in a regular manner between reactivities above and below this reference. **Prompt critical** is, however, a convenient condition for marking the transition from delayed neutron to prompt neutron time scales. A reactor whose reactivity even approaches **prompt critical** is likely to suffer damage due to the rapid rise in power to a very high level. For example, a reactor which has gone prompt critical could experience a several thousand percent power increase in less than one second.

Because the prompt critical condition is so important, a specific unit of reactivity has been defined that relates to it. The unit of reactivity is the dollar (**$**), where one dollar of reactivity is equivalent to the effective delayed neutron fraction . A reactivity unit related to the dollar is the **cent**, where one **cent** is one-hundredth of a dollar. **If the reactivity of the core is one dollar, the reactor is prompt critical**. Because the effective delayed neutron fraction is dependent upon the nuclides used as fuel, the value of the dollar is also dependent on the nuclides used as fuel.

2.5 Stable Period Equation

For normal reactor operating conditions, the value of positive reactivity in the reactor is never permitted to approach the effective delayed neutron fraction, and the reactor period equation is normally written as follows:

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{\lambda_{eff}\,\rho - (d\rho/dt)}$$

where: τ = Reactor Period
β_{eff} = effective delayed neutron fraction
ρ = reactivity
λ_{eff} = effective delayed neutron precursor decay constant
$d\rho/dt$ = rate of change of reactivity

This equation is referred to as the **transient period equation** since it incorporates the term ($d\rho/dt$) to account for the changing amount of reactivity in the core. The ℓ^*/ρ term (prompt period) is normally negligible with respect to the remainder of the equation and is often not included. For conditions when the amount of reactivity in the core is constant ($d\rho/dt = 0$), and the reactor period is unchanging, the equation above can be simplified further to the following form, which is known as the **stable period equation**.

$$\tau = \frac{\bar{\beta}_{eff} - \rho}{}$$

$$\lambda_{eff}\,\rho$$

where: τ = Reactor Period
β_{eff} = effective delayed neutron fraction
ρ = reactivity
λ_{eff} = effective delayed neutron precursor decay constant

2.6 Reactor Startup Rate (SUR)

The **reactor startup rate** (**SUR**) is defined as the number of factors of ten that power changes in one minute. The units of **SUR** are powers of ten per minute, or **decades per minute** (**DPM**). The following equation shows the relationship between **reactor power** and **startup rate**:

$$\mathbf{P = P_0\ x\ 10^{SUR(t)}}$$

where:

P = transient reactor power
$\mathbf{P_0}$ = initial reactor power
SUR = reactor start-up rate (Decades per minute (DPM))
t = time during the reactor transient (minutes)

The relationship between reactor period and startup rate can be developed by considering the following equation;

$$\mathbf{P = P_0\ x\ e^{t/\tau}}$$

where:

τ = reactor period (seconds)
t = time during the reactor transient (seconds)

and

$$\mathbf{P = P_0\ x\ 10^{SUR(t)}}$$

Yields:

$$\mathbf{\frac{P}{P_0} = e^{t/\tau} = 10^{SUR(t)}}$$

Changing the base of the exponential term on the right side of this equation to the natural logarithm "**e**" (**10 = $e^{2.303}$**) and solving the result yields the following:

$$\mathbf{e^{t(sec)/\tau} = e^{2.303SUR(t(min))}}$$

therefore:

$$\mathbf{t(sec)/\tau = 2.303SUR(t(min))}\ \text{and}$$

$$\mathbf{60/\tau = 2.303SUR}$$

$$\mathbf{SUR = 26.06/\tau}$$

2.7 Doubling Time

Sometimes it is useful to discuss the rate of change of reactor power in terms similar to those used in radioactive decay calculations. **Doubling or halving time** are terms that relate to the amount of time it takes reactor power to double or be reduced to one-half the initial power level. If the stable reactor period is known, doubling time can be determined as follows:

Doubling Time (DT) = τ (ln 2)

where: τ = stable reactor period
ln 2 = natural logarithm of 2

When the doubling time is known, the power level change from $\mathbf{P_0}$ is given by the following equation:

$$\mathbf{P = P_0 \ x \ e^{t/DT}}$$

where:

P = transient reactor power
$\mathbf{P_0}$ = initial reactor power
DT = reactor doubling time (seconds)
t = time during the reactor transient (seconds)

SECTION THREE: REACTOR OPERATION

It is important to understand the principles that determine how a reactor responds during all modes of operation. Special measures must be taken during the startup of a reactor to ensure that expected responses are occurring. During power operation, control of the flux shape is necessary to ensure operation within limits and maximum core performance. Even when a reactor is shut down, the fact that the fission products created by the fission process continue to generate heat results in a need to monitor support systems to ensure adequate cooling of the core.

3.1 Startup

When a reactor is started up with un-irradiated fuel, or on those occasions when the reactor is restarted following a long shutdown period, the source neutron population will be very low. In some reactors, the neutron population is frequently low enough that it cannot be detected by the nuclear instrumentation during the approach to criticality.

Installed neutron sources are frequently used to provide a safe, easily monitored reactor startup. The neutron source, together with the sub-critical multiplication process, provides a sufficiently large neutron population to allow monitoring by the nuclear instruments throughout the startup procedure. Continued withdrawal of control rods at this point could cause reactor power to rise at an uncontrollable rate before neutron level first becomes visible on the nuclear instruments.

An alternative to using a startup source is to limit the rate of rod withdrawal, or require waiting periods between rod withdrawal increments. By waiting between rod withdrawal increments, the neutron population is allowed to increase through **sub-critical multiplication**. **Sub=critical multiplication** is the process where source neutrons are used to sustain the chain reaction in a reactor with a multiplication factor ($\mathbf{k_{eff}}$) of less than one. The chain reaction is not "self-sustaining," but if the neutron source is of sufficient magnitude, it compensates for the neutrons lost through absorption and leakage. This process can result in a constant, or increasing, neutron population even though $\mathbf{k_{eff}}$ is less than one.

3.2 Estimated Critical Position

The **1/M** plots, previously discussed, are useful for monitoring the approach to criticality and predicting when criticality will occur based on indications received while the startup is actually in progress. Before the reactor startup is initiated, the operator calculates an estimate of the amount of rod withdrawal that will be necessary to achieve criticality. This process provides an added margin of safety because a large discrepancy between actual and estimated critical rod positions would indicate that the core was not performing as designed.

Depending upon a reactor's design or age, the buildup of **Xenon** within the first several hours following a reactor shutdown may introduce enough negative reactivity to cause the reactor to remain shutdown even with the control rods fully withdrawn. In this situation it is important to be able to predict whether criticality can be achieved, and if criticality cannot be achieved, the startup should not be attempted. For a given set of conditions (such as time since shutdown, temperature, pressure, fuel burnup, **samarium** and **xenon poisoning**) there is only one position of the control rods (and boron concentrations for a reactor with chemical shim) that results in criticality, using the normal rod withdrawal sequence. Identification of these conditions allows accurate calculation of control rod position at criticality. The calculation of an **estimated critical position** (**ECP)** is simply a mathematical procedure that takes into account all of the changes in factors that significantly affect reactivity that have occurred between the time of reactor shutdown and the time that the reactor is brought critical again.

For most reactor designs, the only factors that change significantly after the reactor is shut down are the average reactor temperature and the concentration of fission product poisons. The reactivities normally considered when calculating an **ECP** include the following:

- **Basic Reactivity of the Core:** The reactivity associated with the critical control rod position for a **xenon**-free core at normal operating temperature. This reactivity varies with the age of the core (amount of fuel burnup).

- **Direct Xenon Reactivity:** The reactivity related to the **Xenon** that was actually present in the core at the time it was shutdown. This reactivity is corrected to allow for **Xenon** decay.
- **Indirect Xenon Reactivity:** The reactivity related to the **Xenon** produced by the decay of **Iodine** that was present in the core at the time of shutdown.
- **Temperature Reactivity:** The reactivity related to the difference between the actual reactor temperature during startup and the normal operating temperature.

To arrive at an **ECP (Estimated Critical Position)** of the control rods, the basic reactivity, direct and indirect **Xenon reactivity**, and **temperature reactivity** are combined algebraically to determine the amount of positive control rod reactivity that must be added by withdrawing control rods to attain criticality. A graph of control rod worth versus rod position is used to determine the estimated critical position.

3.3 Core Power Distribution

In order to ensure predictable temperatures and uniform depletion of the fuel installed in a reactor, numerous measures are taken to provide an even distribution of flux throughout the power producing section of the reactor. This shaping, or flattening, of the neutron flux is normally achieved through the use of **reflectors** that affect the flux profile across the core, or by the installation of poisons to suppress the neutron flux where desired. The last method, although effective at shaping the flux, is the least desirable since it reduces neutron economy by absorbing the neutrons.

A reactor core is frequently surrounded by a "reflecting" material to reduce the ratio of peak flux to the flux at the edge of the core fuel area. Reflector materials are normally not fissionable, have a high scattering cross section, and have a low absorption cross section. Essentially, for thermal reactors a good moderator is a good reflector. Water, heavy water, beryllium, zirconium, or graphite are commonly used as reflectors. In fast reactor systems, reflectors are not composed of moderating materials because it is desired to keep neutron energy high. The reflector functions by scattering some of the neutrons, which would have leaked from a bare (unreflected) core, back into the fuel to produce additional fissions.

the general effect of reflection in the thermal reactor system where core power is proportional to the thermal flux is to raise the power density of the core periphery and thus increase the core average power level without changing the peak power. the thermal flux in the reflector may actually be higher than that in the outermost fuel since there are very few absorptions in the reflector.

Varying the fuel enrichment or fuel concentrations in the core radially, axially, or both, can readily be used to control power distribution. Varying fuel

concentrations or poison loading for flux shaping is frequently referred to as zoning.

3.4 Power Tilt

A **power tilt**, or **flux tilt**, is a specific type of core power distribution problem. It is a non-symmetrical variation of core power in one quadrant of the core relative to the others. The power in one portion might be suppressed by over-insertion of control rods in that portion of the core, which, for a constant overall power level, results in a relatively higher flux in the remainder of the core. This situation can lead to xenon oscillations, which were previously discussed.

3.5 Shutdown Margin

Shutdown margin is the instantaneous amount of reactivity by which a reactor is sub-critical or would be sub-critical from its present condition assuming all control rods are fully inserted except for the single rod with the highest integral worth, which is assumed to be fully withdrawn. Shutdown margin is required to exist at all times, even when the reactor is critical. It is important that there be enough negative reactivity capable of being inserted by the control rods to ensure complete shutdown at all times during the core lifetime.

A shutdown margin in the range of one to five percent reactivity is typically required. The stuck rod criterion refers to the fact that the shutdown margin does not take credit for the insertion of the highest worth control rod. The application of the stuck rod criterion ensures that the failure of a single control rod will not prevent the control rod system from shutting down the reactor.

3.6 Operation

During reactor operation, numerous parameters such as temperature, pressure, power level, and flow are continuously monitored and controlled to ensure safe and stable operation of the reactor. The specific effects of variations in these parameters vary greatly depending upon reactor design, but generally the effects for thermal reactors are as follows.

A. Temperature

The most significant effect of a variation in temperature upon reactor operation is the addition of positive or negative reactivity. As previously discussed, reactors are generally designed with **negative temperature coefficients of reactivity** (moderator and fuel temperature coefficients) as a self-limiting safety feature. A rise in reactor temperature results in the addition of **negative reactivity**.

If the rise in temperature is caused by an increase in reactor power, the negative reactivity addition slows, and eventually turns the increase in reactor power. This is a highly desirable effect because it provides a negative feedback in the event of an undesired power excursion.

Negative temperature coefficients can also be utilized in water cooled and moderated power reactors to allow reactor power to automatically follow energy demands that are placed upon the system. For example, consider a reactor operating at a stable power level with the heat produced being transferred to a heat exchanger for use in an external closed cycle system. If the energy demand in the external system increases, more energy is removed from reactor system causing the temperature of the reactor coolant to decrease. As the reactor temperature decreases, positive reactivity is added and a corresponding increase in reactor power level results. As reactor power increases to a level above the level of the new energy demand, the temperature of the moderator and fuel increases, adding negative reactivity and decreasing reactor power level to near the new level required to maintain system temperature.

Some slight oscillations above and below the new power level occur before steady state conditions are achieved. The final result is that the average temperature of the reactor system is essentially the same as the initial temperature, and the reactor is operating at the new higher required power level. The same inherent stability can be observed as the energy demand on the system is decreased.

B. Pressure

The pressure applied to the reactor system can also affect reactor operation by causing changes in reactivity. The reactivity changes result from changes in the density of the moderator in response to the pressure changes. For example, as the system pressure rises, the moderator density increases and results in greater moderation, less neutron leakage, and therefore the insertion of positive reactivity. A reduction in system pressure results in the addition of negative reactivity. Typically, in pressurized water reactors (**PWR**), the magnitude of this effect is considerably less than that of a change in temperature. In two-phase systems such as boiling water reactors (**BWR**), however, the effects of pressure changes are more noticeable because there is a greater change in moderator density for a given change in system pressure.

C. Power Level

A change in reactor power level can result in a change in reactivity if the power level change results in a change in system temperature. The power level at which the reactor is producing enough energy to make up for the energy lost to ambient is commonly referred to as **the point of adding heat.** If a reactor is operating well below the point of adding heat, then variations in power level produce no measurable variations in temperature. At power levels above the point of adding heat, temperature varies with power level, and the reactivity changes will follow the convention previously described for temperature variations.

The inherent stability and power turning ability of a negative temperature coefficient are ineffective below the point of adding heat. If a power excursion is initiated from a very low power level, power will continue to rise unchecked until the point of adding heat is reached, and the subsequent temperature rise adds negative reactivity to slow, and turn, the rise of reactor power. In this region, reactor safety is provided by automatic reactor shutdown systems and operator action.

D. Flow

At low reactor power levels, changing the flow rate of the coolant through the reactor does not result in a measurable reactivity change because fuel and moderator temperatures and the fraction of steam voids occurring in the core are not changed appreciably. When the flow rate is varied, however, the change in temperature that occurs across the core (outlet versus inlet temperature) will vary inversely with the flow rate. At higher power levels, on liquid cooled systems, increasing flow will lower fuel and coolant temperatures slightly, resulting in a small positive reactivity insertion. A positive reactivity addition also occurs when flow is increased in a two-phase (steam-water) cooled system. Increasing the flow rate decreases the fraction of steam voids in the coolant and results in a positive reactivity addition. This property of the moderator in a two-phase system is used extensively in commercial BWRs. Normal power variations required to follow load changes on BWRs are achieved by varying the coolant/moderator flow rate.

3.7 Core Burnup

As a reactor is operated, atoms of fuel are constantly consumed, resulting in the slow depletion of the fuel frequently referred to as **core burnup**. There are several major effects of this fuel depletion. The first, and most obvious, effect of the fuel burnup is that the control rods must be withdrawn or chemical shim concentration reduced to compensate for the negative reactivity effect of this burnup.

Some reactor designs incorporate the use of supplemental burnable poisons in addition to the control rods to compensate for the reactivity associated with excess fuel in a new core. These fixed burnable poisons burn out at a rate that approximates the burnout of the fuel and they reduce the amount of control rod movement necessary to compensate for fuel depletion early in core life.

As control rods are withdrawn to compensate for fuel depletion, the effective size of the reactor is increased. By increasing the effective size of the reactor, the probability that a neutron slows down and is absorbed while it is still in the reactor is also increased. Therefore, neutron leakage decreases as the effective reactor size is increased.

The magnitude of the moderator negative temperature coefficient is determined in part by the change in neutron leakage that occurs as the result of a change in moderator temperature. Since the fraction of neutrons leaking out is less with the larger core, a given temperature change will have less of an effect on the leakage. Therefore, the magnitude of the moderator negative temperature coefficient decreases with fuel burnup.

There is also another effect that is a consideration only on reactors that use dissolved boron in the moderator (chemical shim). As the fuel is burned up, the dissolved boron in the moderator is slowly removed (concentration diluted) to compensate for the negative reactivity effects of fuel burnup. This action results in a larger (more negative) moderator temperature coefficient of reactivity in a reactor using chemical shim. This is due to the fact that when water density is decreased by rising moderator temperature in a reactor with a negative temperature coefficient, it results in a negative reactivity addition because some moderator is forced out of the core. With a coolant containing dissolved poison, this density decrease also results in some poison being forced out of the core, which is a positive reactivity addition, thereby reducing the magnitude of the negative reactivity added by the temperature increase. Because as fuel burnup increases the concentration of boron is slowly lowered, the positive reactivity added by the above poison removal process is lessened, and this results in a larger negative temperature coefficient of reactivity. The following effect of fuel burnup is most predominant in a reactor with a large concentration of uranium-238. As the fission process occurs in a thermal reactor with low or medium enrichment, there is some conversion of uranium-238 into plutonium-239. Near the end of core life in certain reactors, the power contribution from the fission of plutonium-239 may be comparable to that from the fission of uranium-235. The value of the delayed neutron fraction (b) for uranium-235 is 0.0064 and for plutonium-239 is 0.0021. Consequently, as core burnup progresses, the effective delayed neutron fraction for the fuel decreases appreciably. It follows then that the amount of reactivity insertion needed to produce a given reactor period decreases with burnup of the fuel.

3.8 Shutdown

A reactor is considered to be shut down when it is sub-critical and sufficient shutdown reactivity exists so there is no immediate probability of regaining criticality. Shutdown is normally accomplished by insertion of some (or all) of the control rods, or by introduction of soluble neutron poison into the reactor coolant.

The rate at which the reactor fission rate decays immediately following shutdown is similar for all reactors provided a large amount of negative reactivity is inserted. After a large negative reactivity addition the neutron level undergoes a rapid decrease of about two decades (prompt drop) until it is at the level of production of delayed neutrons. Then the neutron level slowly

drops off as the delayed neutron precursors decay, and in a short while only the longest-lived precursor remains in any significant amount.

This precursor determines the final rate of decrease in reactor power until the neutron flux reaches the steady state level corresponding to the sub-critical multiplication of the neutron source. The half-life of the longest lived delayed neutron precursor results in a reactor period of around -80 seconds or a startup rate of -1/3 **DPM** for most reactors after a reactor shutdown. One noticeable exception to this is a heavy water reactor. In a heavy water reactor, the photoneutron source is extremely large after shutdown due to the amount of deuterium in the moderator and the large number of high energy gammas from short-lived fission product decay. The photo-neutron source is large enough to have a significant impact on neutron population immediately after shutdown. The photo-neutron source has the result of flux levels decreasing more slowly so that a heavy water reactor will have a significantly larger negative reactor period after a shutdown. Throughout the process of reactor shutdown the nuclear instrumentation is closely monitored to observe that reactor neutron population is decreasing as expected, and that the instrumentation is functioning properly to provide continuous indication of neutron population. Instrumentation is observed for proper overlap between ranges, comparable indication between multiple instrument channels, and proper decay rate of neutron population.

A distinction should be made between indicated reactor power level after shutdown and the actual thermal power level. The indicated reactor power level is the power produced directly from fission in the reactor core, but the actual thermal power drops more slowly due to decay heat production as previously discussed. Decay heat, although approximately 5 to 6% of the steady state reactor power prior to shutdown, diminishes to less than 1% of the pre-shutdown power level after about one hour. After a reactor is shutdown, provisions are provided for the removal of decay heat. If the reactor is to be shut down for only a short time, operating temperature is normally maintained. If the shutdown period will be lengthy or involves functions requiring cooldown of the reactor, the reactor temperature can be lowered by a number of methods. The methods for actually conducting cooldown of the reactor vary depending on plant design, but in all cases limitations are imposed on the maximum rate at which the reactor systems may be cooled. These limits are provided to reduce the stress applied to system materials, thereby reducing the possibility of stress induced failure. Although a reactor is shut down, it must be continuously monitored to ensure the safety of the reactor. Automatic monitoring systems are employed to continuously collect and assess the data provided by remote sensors. It is ultimately the operator who must ensure the safety of the reactor.

3.9 Decay Heat

About 7 percent of the 200 MeV produced by an average fission is released at some time after the instant of fission. This energy comes from the decay of the fission products. When a reactor is shut down, fission essentially ceases, but decay energy is still being produced. The energy produced after shutdown is referred to as decay heat. The amount of decay heat production after shutdown is directly influenced by the power history of the reactor prior to shutdown. A reactor operated at full power for 3 to 4 days prior to shutdown has much higher decay heat generation than a reactor operated at low power for the same period. The decay heat produced by a reactor shutdown from full power is initially equivalent to about 5 to 6% of the thermal rating of the reactor. This decay heat generation rate diminishes to less than 1% approximately one hour after shutdown. However, even at these low levels, the amount of heat generated requires the continued removal of heat for an appreciable time after shutdown. Decay heat is a long-term consideration and impacts spent fuel handling, reprocessing, waste management, and reactor safety.

END OF CHAPTER FOUR: REACTOR OPERATIONS