Electromagnetic Theory

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Induced Current

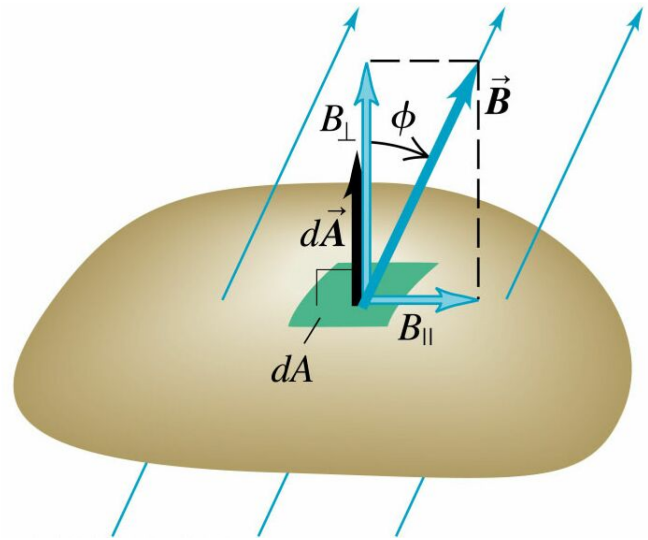
- Past experiments with magnetism have shown the following
  - When a magnet is moved towards or away from a circuit, there is an induced current in the circuit
  - This is still true even if it is the circuit that is moved towards or away from the magnet
  - When both are at rest with respect to each, there is *no* induced current
- The physical reason behind this phenomena

Magnetic Flux

Just as was the case with electric fields, we can define a magnetic flux

$$d\Phi_B = \vec{B} \cdot d\vec{A}$$

where *dA* is an incremental area with the total flux being given by:



$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

Note that this integral is *not* over a closed surface, for that integral would yield zero for an answer, since there are no sources or sinks for the magnetic as there is with electric fields

Faraday’s Law of Induction

It was Michael Faraday who was able to link the induced current with a changing magnetic flux

He stated that:

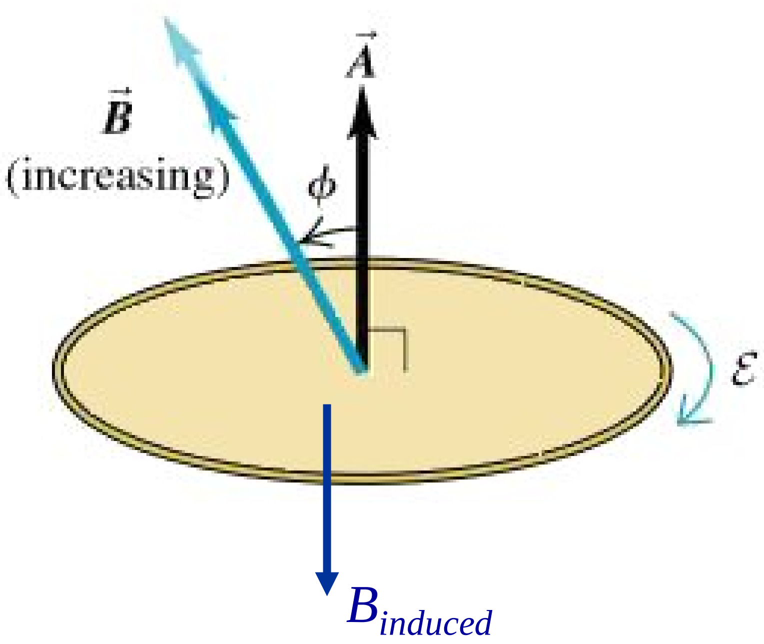
“The induced emf (electromotive force) in a closed loop equals the negative of the time rate of change of the magnetic flux through the loop”

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

The induced emf *opposes* the change that is occurring

Increasing Magnetic Flux

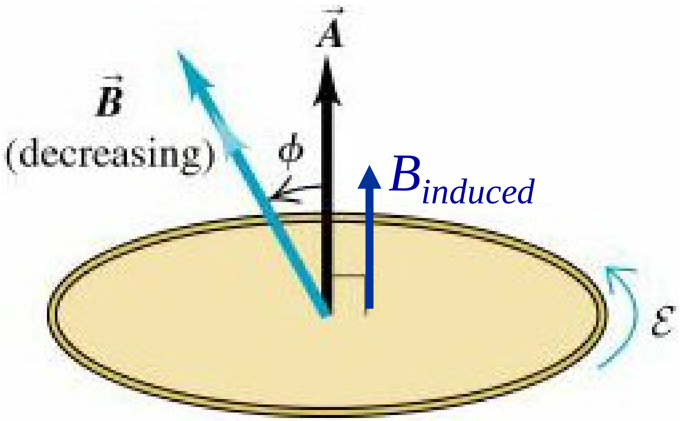
Suppose that we are given a closed circuit through which the magnetic field, *flux*, is increasing, then according to Faraday's Law there will be an induced emf in the loop



The sense of the emf will be so that the induced current will set up a magnetic field that will oppose this increase of external flux

Decreasing Magnetic Flux

Suppose now that the magnetic field is decreasing through the closed loop, Faraday's Law again states that there will be an induced emf in the loop



The sense of the emf will be so that the induced current will set up a magnetic field that will try to keep the magnetic flux at its original value.

Changing Area

Both of the previous examples were based on a changing external magnetic flux

How do we treat the problem if the magnetic field is constant and it is the area that is changing

Basically the same way, it is the changing flux that will be opposed

If the area is increasing, then the flux will be increasing and if the area is decreasing then the flux will also be decreasing

In either case, the induced emf will be such that the change will be opposed

Lenz's Law

All of the proceeding can be summarized as follows

> "The direction of any magnetic induction effect is such as to oppose the cause of the effect"

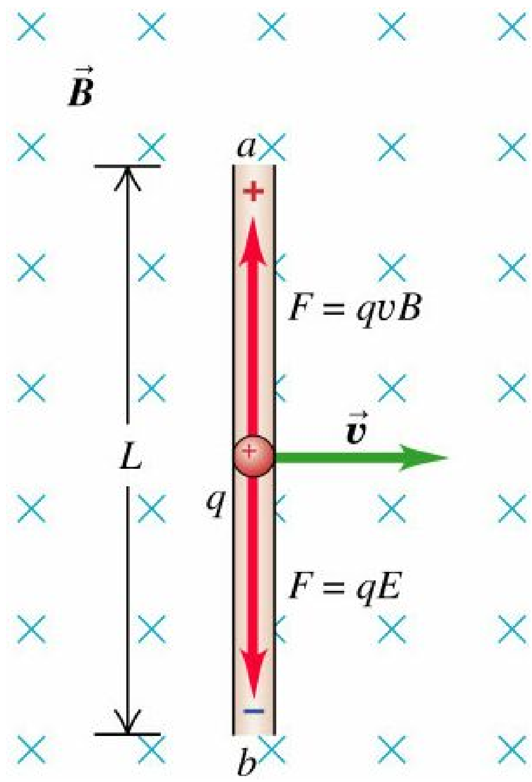
Remember that the cause of the effect could be either a changing external magnetic field, or a changing area, or *both*

$$d\Phi_B = d\vec{B} \cdot \vec{A} + \vec{B} \cdot d\vec{A}$$

Motional EMF: Part One

In previous discussions we had mentioned that a charge moving in a magnetic field experiences a force

Suppose now that we have a conducting rod moving in a magnetic field as shown:



The positive charges will experience a magnetic force upwards, while the negative charges will experience a magnetic force downwards

Charges will continue to move until the magnetic force is balanced by an opposing electric force due to the charges that have already moved

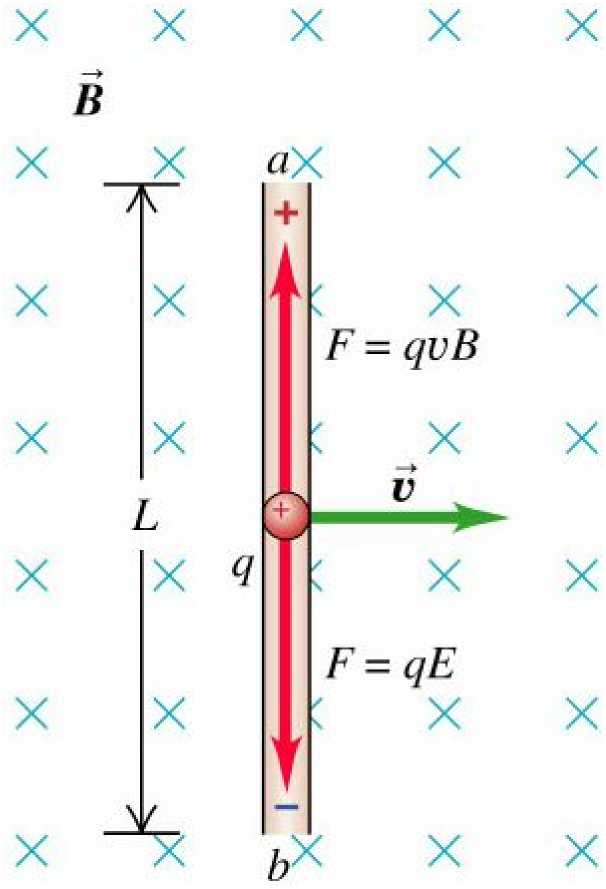
Motional EMF: Part Two

So we then have that $E = vB$

But we also have that $E = V_{ab} / L$

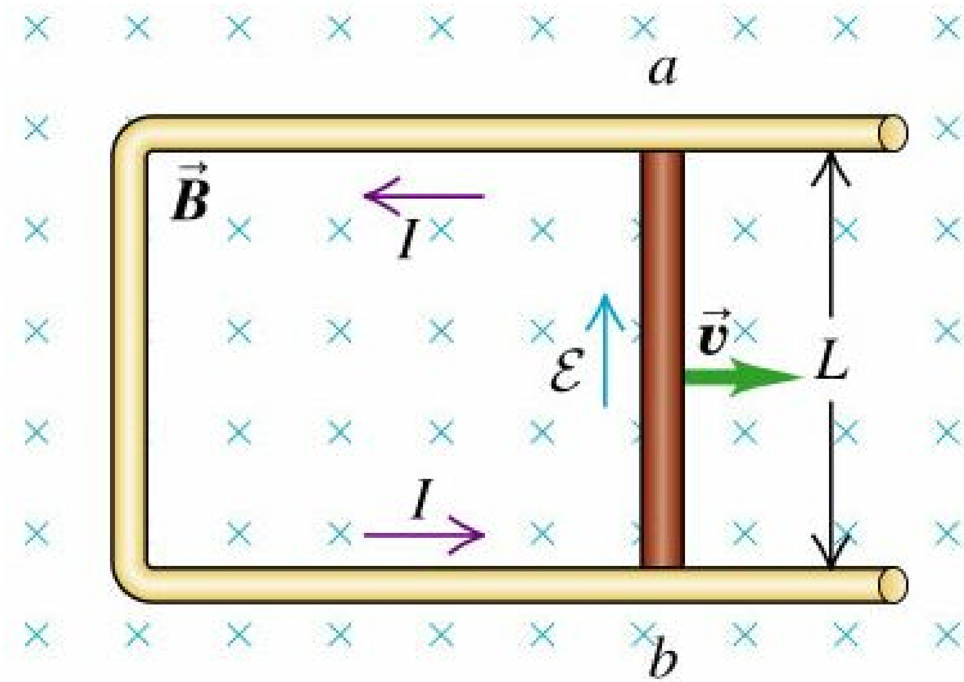
So $V_{ab} = v\,B\,L$

We have an induced potential difference across the ends of the rod



Motional EMF: Part Three

If this rod were part of a circuit, we then would have an induced current



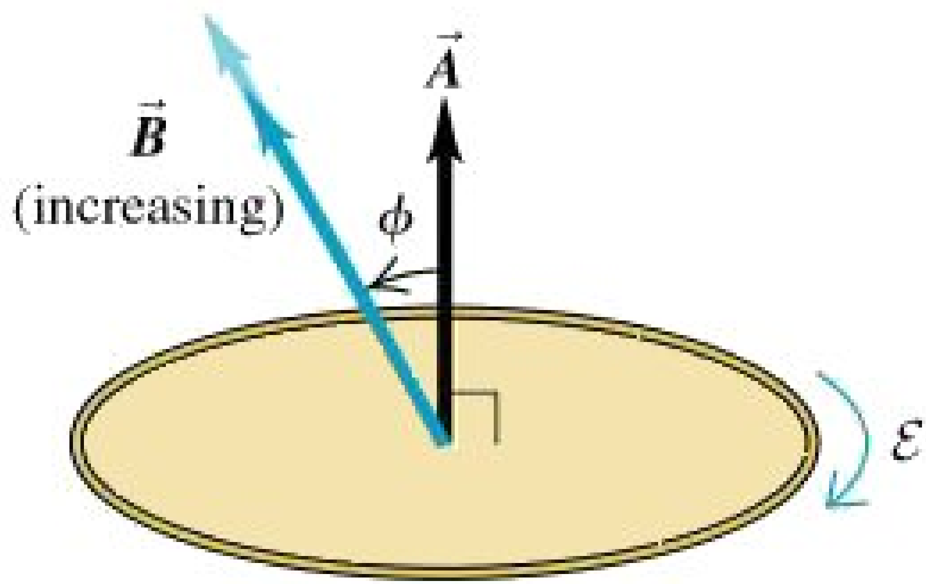
The sense of the induced *emf* can be gotten from Lenz's Law

Induced Electric Fields: Part One

We again look at the closed loop through which the magnetic flux is changing

We now know that there is an induced current in the loop

But what is the force that is causing the charges to move in the loop



It can’t be the magnetic field, as the loop is not moving

Induced Electric Fields: Part Two

The only other thing that could make the charges move would be an electric field that is induced in the conductor

This type of electric field is different from what we have dealt with before

Previously our electric fields were due to charges and these electric fields were conservative

Now we have an electric field that is due to a changing magnetic flux and this electric field is non-conservative

Induced Electric Fields: Part Three

Remember that for conservative forces, the work done in going around a complete loop is *zero*

Here there is a net work done in going around the loop that is given by

$$q\varepsilon$$

But the total work done in going around the loop is also given by

$$q\oint \vec{E}\cdot d\vec{l}$$

Equating these two we then have $\oint \vec{E}\cdot d\vec{l} = \varepsilon$

Induced Electric Fields: Part Four

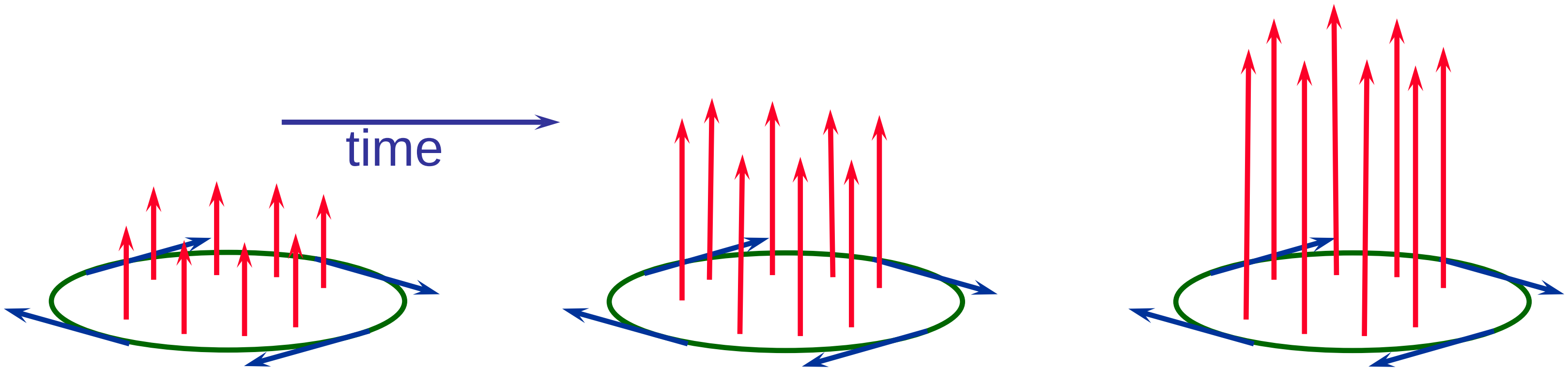
But previously we found that the emf was related to the negative of the time rate of change of the magnetic flux

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

So we then have that: $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$

This is just another way of stating Faraday's Law but for stationary paths

Electro-Motive Force: Part One



A magnetic field, increasing in time, passes through the loop

An electric field is generated "ringing" the increasing magnetic field

Electro-Motive Force: Part Two

Loop integral of E-field is the “emf”:

$$\oint \vec{E} \cdot d\vec{l} = \varepsilon$$

The loop does not have to be a wire - the emf exists even in vacuum!

When we put a wire there, the electrons respond to the emf, producing a current

Displacement Current: Part One

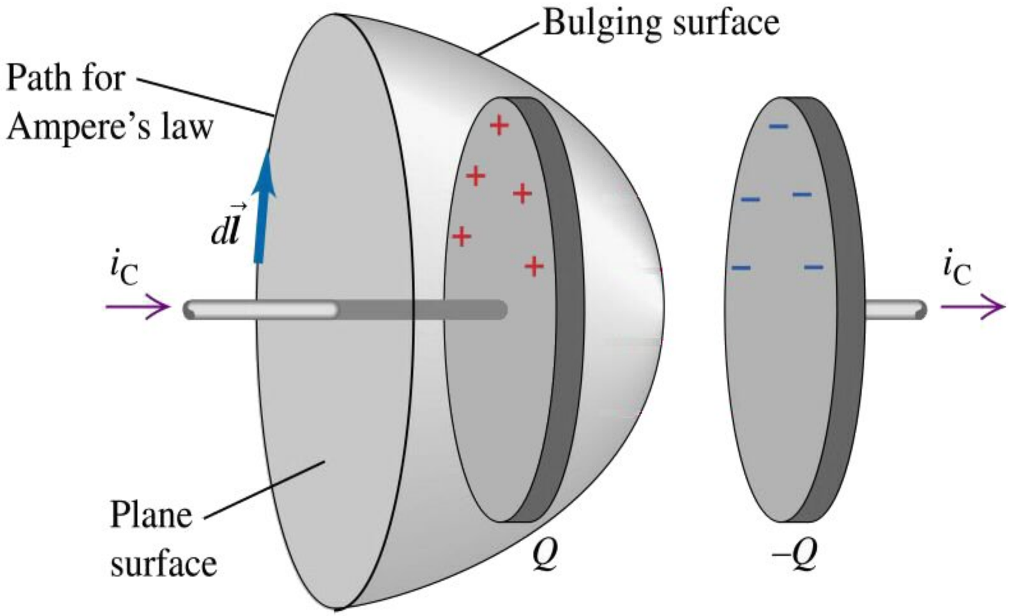
We have used Ampere's Law to calculate the magnetic field due to currents

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

where $I_{enclosed}$ is the current that cuts through the area bounded by the integration path

But in this formulation, Ampere's Law is incomplete

Displacement Current: Part Two



Suppose we have a parallel plate capacitor being charged by a current $I_{c.}$

We apply Ampere's Law to path that is shown.

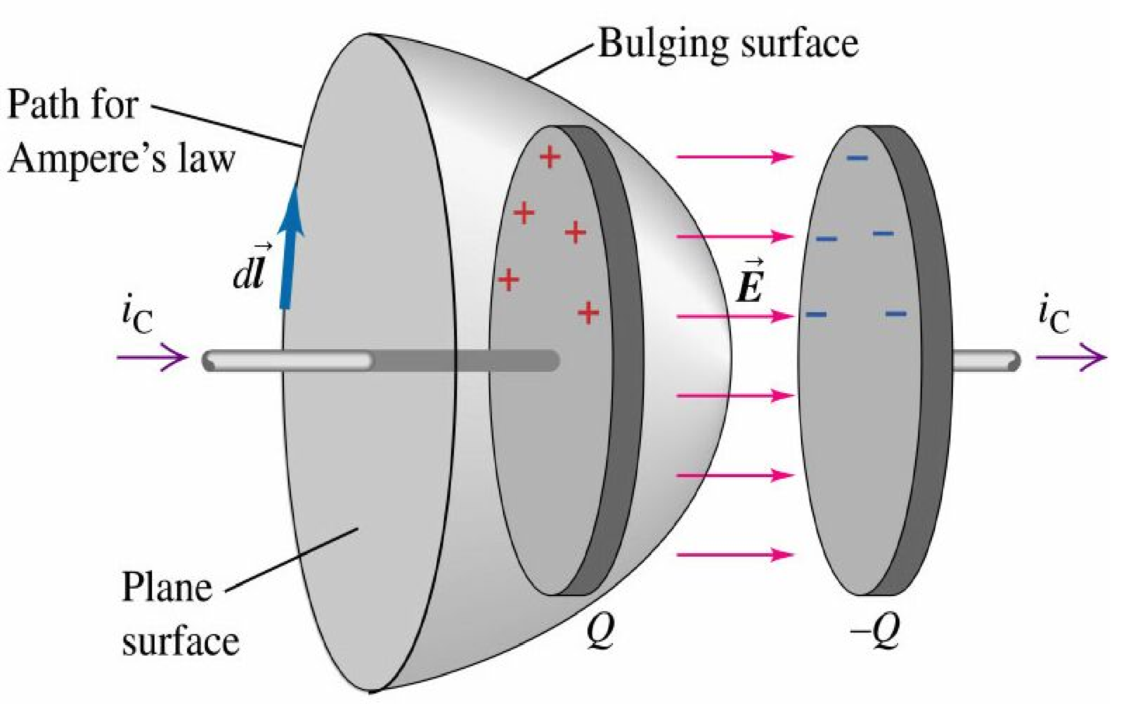
For this path, the integral is just: $\mu_0 I_{enclosed}$

For the plane surface which is bounded by the path, this is just: I_c

Displacement Current: Part Three

But for the bulging surface which is also bounded by the integration path, $I_{enclosed}$ is zero

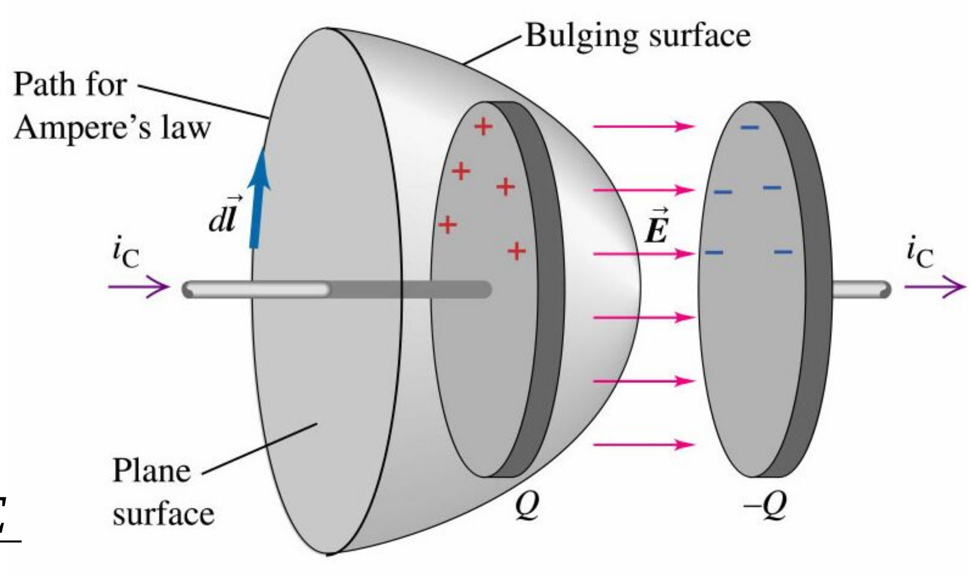
We have a magnetic field and since there is charge on the plates of the capacitor, there is an electric field in the region between the plates



Displacement Current: Part Four

The charge on the capacitor is related to the electric field by

$$q = \varepsilon E A = \varepsilon \Phi_E$$



We define the displacement current, by

$$I_{Displacement} = \frac{dq}{dt} = \varepsilon \frac{d\Phi_E}{dt}$$

We can now rewrite Ampere's Law including this displacement current

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_c + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

Displacement Current

We can now rewrite Ampere's Law including this displacement current

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_c + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

Maxwell's Equations

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \qquad \text{(Gauss's law for } \vec{E}\text{)}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad \text{(Gauss's law for } \vec{B}\text{)}$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}} \qquad \text{(Ampere's law)}$$

Collectively these are known as Maxwell's Equations

Maxwell's Equations

These four equations describe all of classical electric and magnetic phenomena

Faraday's Law links a changing magnetic field with an induced electric field

Ampere's Law links a changing electric field with an induced magnetic field

Further manipulation of Faraday's and Ampere's Laws eventually yield a second order differential equation which is the wave equation with a prediction for the wave speed of

$$\boldsymbol{c} = \frac{1}{\sqrt{\boldsymbol{\mu}_0 \boldsymbol{\varepsilon}_0}} = 3 \times 10^8 \ \boldsymbol{m} / \sec = \textbf{Speed of Light}$$

Maxwell's Equations: Part One

$$\vec{\nabla} \cdot \vec{E} = 0 \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \mu\varepsilon \frac{\partial \vec{E}}{\partial t}$$

where $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field, ε is the permittivity, and μ is the permeability of the medium.

As written, they assume no charges (or free space).

Maxwell's Equations: Part Two

Take $\vec{\nabla}\times$ of: $$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{E}]=\vec{\nabla}\times[-\frac{\partial\vec{B}}{\partial t}]$$

Change the order of differentiation on the RHS:

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{E}]=-\frac{\partial}{\partial t}[\vec{\nabla}\times\vec{B}]$$

Maxwell's Equations: Part Three

But:

$$\vec{\nabla} \times \vec{B} = \mu\varepsilon \frac{\partial \vec{E}}{\partial t}$$

Substituting for $\vec{\nabla} \times \vec{B}$, we have:

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\frac{\partial}{\partial t}[\vec{\nabla} \times \vec{B}] \Rightarrow \vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\frac{\partial}{\partial t}[\mu\varepsilon \frac{\partial \vec{E}}{\partial t}]$$

Or:

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

assuming that μ and ε are constant in time.

Maxwell's Equations: Part Four

Identity: $$\vec{\nabla}\times[\vec{\nabla}\times\vec{f}\,] \equiv \vec{\nabla}(\vec{\nabla}\cdot\vec{f}\,)-\nabla^2\vec{f}$$

Using the identity, $$\vec{\nabla}\times[\vec{\nabla}\times\vec{E}]=-\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

becomes: $$\vec{\nabla}(\vec{\nabla}\cdot\vec{E})-\nabla^2\vec{E}=-\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

If we now assume zero charge density: $\rho = 0$, then

$$\vec{\nabla}\cdot\vec{E} = 0$$

and we're left with the Wave Equation!

$$\nabla^2\vec{E}=\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} \qquad \text{where } \mu\varepsilon \equiv 1/c^2$$

Why light waves are transverse

Suppose a wave propagates in the x-direction. Then it's a function of x and t (and not y or z), so all y- and z-derivatives are zero:

$$\frac{\partial E_y}{\partial y} = \frac{\partial E_z}{\partial z} = \frac{\partial B_y}{\partial y} = \frac{\partial B_z}{\partial z} = 0$$

Now, in a charge-free medium, $\vec{\nabla} \cdot \vec{E} = 0 \quad and \quad \vec{\nabla} \cdot \vec{B} = 0$

that is, $\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \qquad \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$

Substituting the zero values, we have: $\frac{\partial E_x}{\partial x} = 0 \text{ and } \frac{\partial B_x}{\partial x} = 0$

So the longitudinal fields are at most **constant**, and not waves.

Why light waves are transverse

Suppose a wave propagates in the x-direction and has its electric field along the y-direction [so $\boldsymbol{E_x} = \boldsymbol{E_z} = \mathbf{0}$, and].
What is the direction of the magnetic field?

Use: $$-\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \vec{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}, \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right)$$

So: $$-\frac{\partial \vec{B}}{\partial t} = \left(0, 0, \frac{\partial E_y}{\partial x} \right)$$

In other words: $$-\frac{\partial B_z}{\partial t} = \frac{\partial E_y}{\partial x}$$

And the magnetic field points in the z-direction.

The magnetic-field strength in a light wave

Suppose a wave propagates in the x-direction and has its electric field in the y-direction. What is the strength of the magnetic field?

Start with: $-\dfrac{\partial B_z}{\partial t} = \dfrac{\partial E_y}{\partial x}$ and $E_y\left(\vec{r},t\right) = \underset{\sim}{E}_0 \exp\left[i\left(kx - \omega t\right)\right]$

We can integrate: $$B_z(x,t) = B_z(x,0) - \int_0^t \frac{\partial E_y}{\partial x}\,dt$$

Take $B_z(x,0) = 0$

Differentiating E_y with respect to x yields an ik, and integrating with respect to t yields a $1/\text{-}i\omega$.

So: $$B_z(x,t) = -\frac{ik}{-i\omega}\underset{\sim}{E}_0 \exp\left[i(kx - \omega t)\right]$$

But $\omega / k = c$: $$B_z(x,t) = \frac{1}{c} E_y(x,t)$$

Electromagnetic Spectrum

The Electromagnetic Spectrum

We give different names to different "parts" of the electromagnetic spectrum. These "parts" are separated according to wavelength.

These names are very familiar to you.

Your eyes are sensitive to only the *very tiny* part of the spectrum which we call "visible light".

Your eyes are most sensitive to green and yellow light. Your eyes are not very sensitive to red and blue light.

Electromagnetic Spectrum: Part One

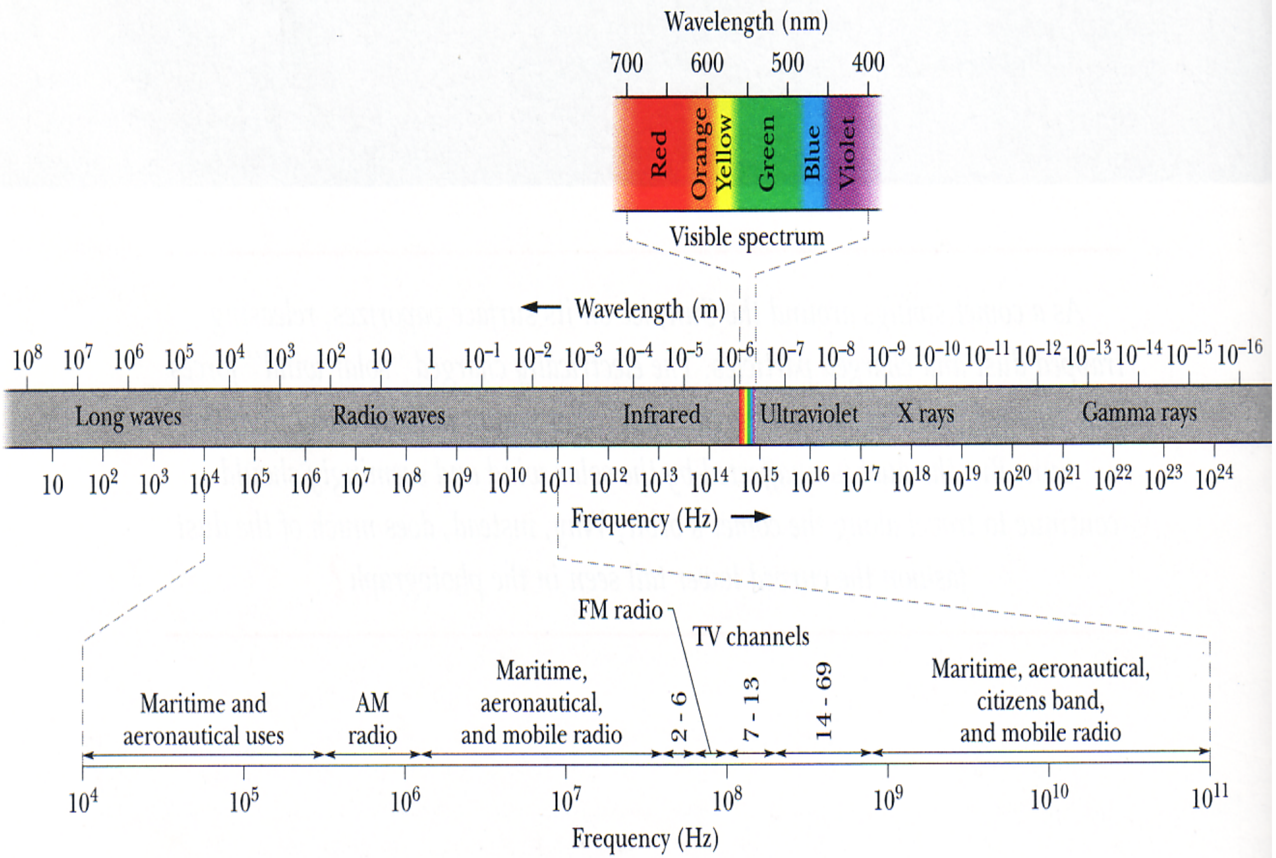
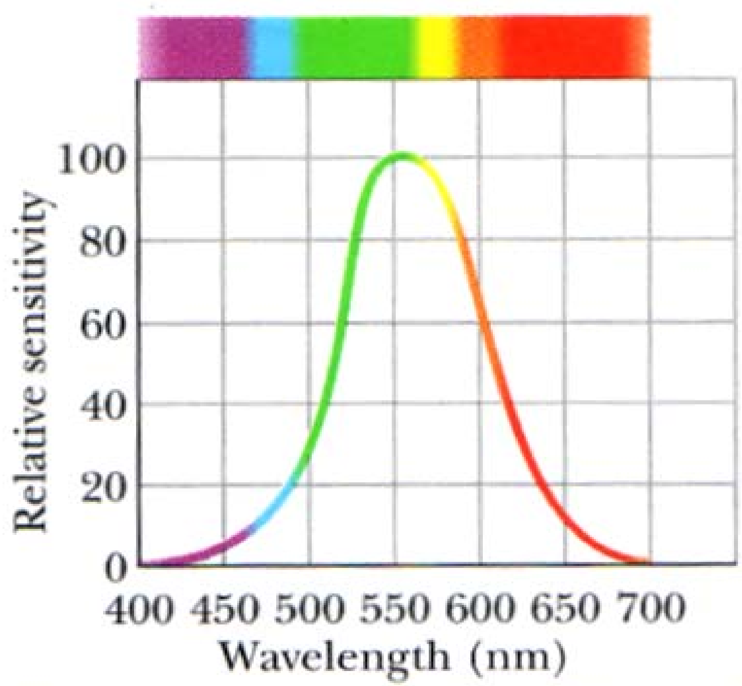


FIGURE 34-1 The electromagnetic spectrum.

Electromagnetic Spectrum: Part Two



URE 34-2 The relative sensitivity of the eye to electromag- waves at different wavelengths. This portion of the electro- netic spectrum consists of *visible light*.

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Electromagnetic Waves: Part Two

Electromagnetic Waves

A changing electric field gives rise to a changing magnetic field, which gives rise to a changing electric field, which gives rise to a changing magnetic field, which ...

$\Rightarrow$ You don't need any charges or currents around to produce "waving" E and B fields.

All electromagnetic waves move at the speed

$$c = 299792458 \text{ m/sec}$$

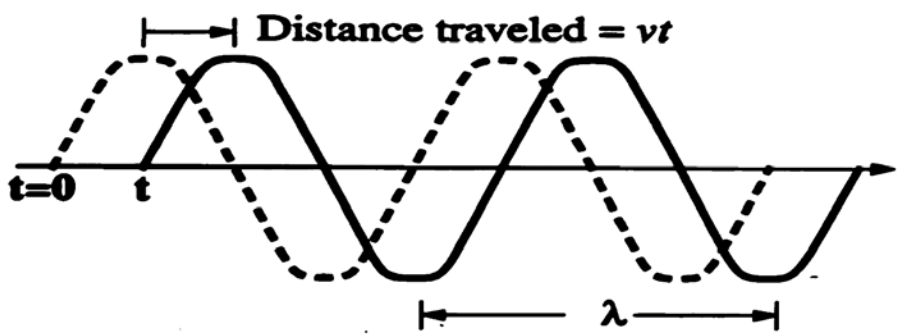
The "speed of light"

$$c = \lambda \times f$$

Electromagnetic Waves

Properties of Waves

$$y(x,t) = y_m \sin(kx - \omega t)$$



Oscillates in *time* with *period* $T = 2\pi/\omega$

Frequency f defined as $f = 1/T$

Oscillates in *space* with *wavelength* $\lambda = 2\pi/k$

Moves a distance x in a time t with a *speed*

$$v = \frac{x}{t} = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

$\Rightarrow$ Speed of wave = wavelength × frequency

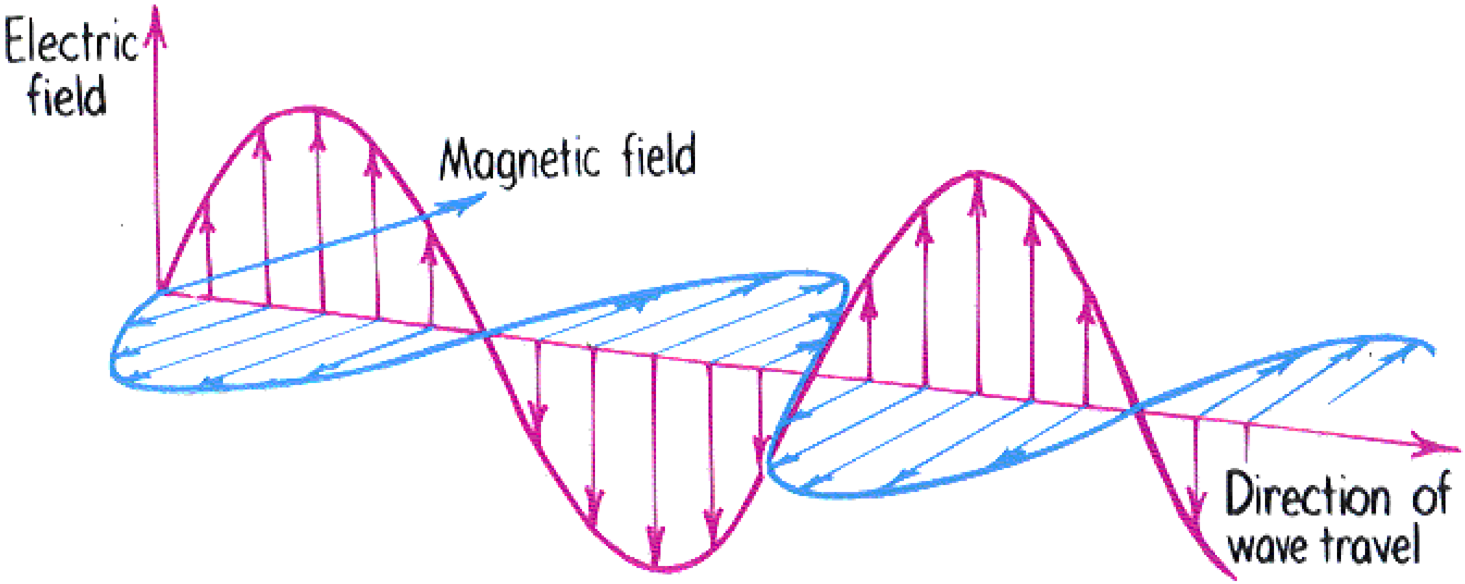
Electromagnetic Waves

- $E = E_m \sin(kx - \omega t)$
- $B = B_m \sin(kx - \omega t)$

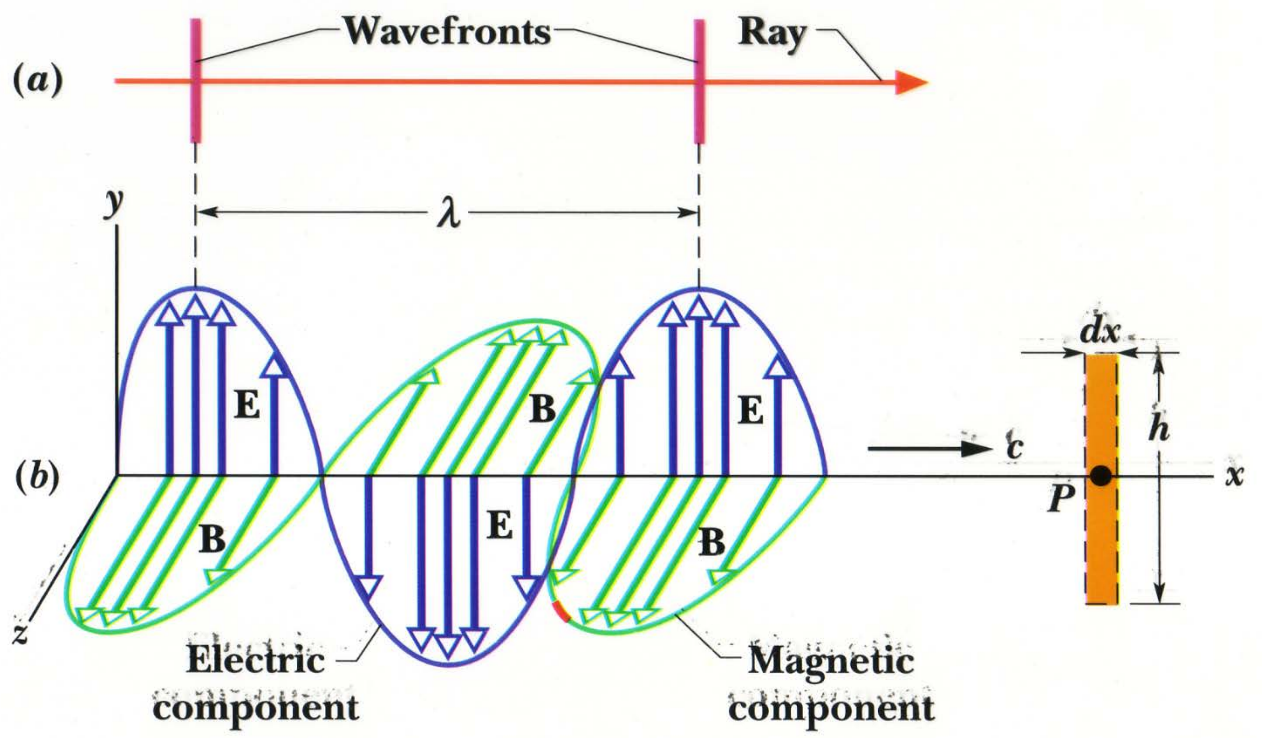
$$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f = \frac{2\pi}{T}$$

$$\text{wave speed} = c = \frac{\omega}{k} = \lambda f$$

Electromagnetic Field Propagation



Electromagnetic Field Propagation



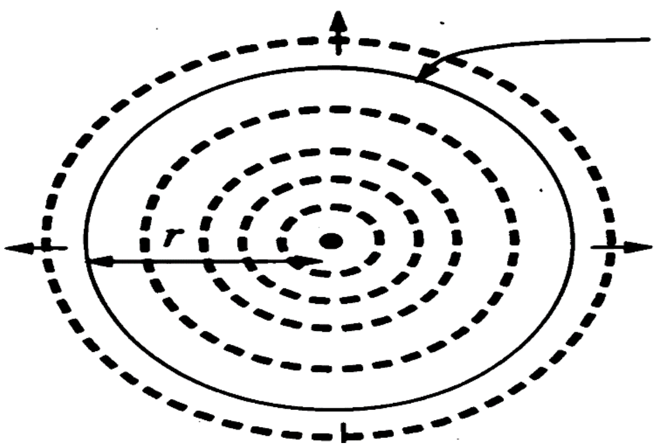
EM Field Intensity

Intensity of EM Radiation

Electromagnetic waves transport *energy*.

The amount of power (i.e. energy per second) transported by an electromagnetic wave per unit surface area is called "intensity" I.

Consider a source which emits radiation power P equally in all directions (i.e. "isotropic"):



Power is distributed uniformly over a sphere of radius r

$$I = \frac{P}{4\pi r^2}$$

Poynting Vector

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$$

- This is a measure of power per area. Units are watts per meter^2.
- Direction is the direction in which the wave is moving.

Poynting Vector

- However, since **E** and **B** are perpendicular,

$$S = \frac{1}{\mu_0} EB$$

and since $\dfrac{E}{B} = c$

$$S = \frac{1}{c\mu_0} E^2 = \frac{c}{\mu_0} B^2$$

Intensity

$$S = \frac{1}{c\mu_0} E_m^2 \sin^2(kx - \omega t)$$

$$\bar{S} = \text{Intensity} = I = \frac{1}{2c\mu_0} E_m^2$$

$$\bar{S} = I = \frac{1}{c\mu_0} E_{rms}^2$$

$$E_{rms} = \frac{E_m}{\sqrt{2}}$$

Radiation Pressure

Electromagnetic radiation transports energy.

It also transports *momentum*. This means it can exert a force. This force is called “radiation pressure”.

Example: Comet tails

Radiation Pressure

$$F = \frac{\Delta p}{\Delta t} = \frac{1}{c}\frac{\Delta U}{\Delta t} = \frac{1}{c} IA$$

$$F = \frac{IA}{c} \quad \text{(complete absorption)}$$

$$F = \frac{2IA}{c} \quad \text{(complete reflection)}$$

Radiation Pressure

$$\text{Pressure} = \frac{F}{A}$$

$$p_r = \frac{I}{c} \quad \text{(complete absorption)}$$

$$p_r = \frac{2I}{c} \quad \text{(complete reflection)}$$

Radiation Pressure

Momentum:

$$\Delta p = \frac{\Delta U}{c} \quad \text{(complete absorption)}$$

$$\Delta p = \frac{2\Delta U}{c} \quad \text{(complete reflection)}$$

Maxwell's Equations

So far we have obtained Maxwell's four equations:

$$\nabla \cdot \mathbf{E} = \frac{\rho_f + \rho_b}{\epsilon_0};$$

$$\nabla \cdot \mathbf{B} = \mathbf{0};$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial \mathbf{t}};$$

$$\nabla \times \mathbf{B} = \mu_0(\mathbf{J_f} + \mathbf{J_e}); \quad \mathbf{(steady)}$$

Generally?

AMPERE'S LAW

Original:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i$$

As modified by Maxwell:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \left(i + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

Symmetry

Maxwell's Equations in Free Space

$$q = 0 \qquad\qquad i = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = 0 \qquad\qquad \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0 \qquad\qquad \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Continuity

With Maxwell's modification:

At P_1: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i$

At P_2: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \dfrac{d\Phi_E}{dt} = \mu_0 i_d$

At P_3: $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i$

$$\varepsilon_0 \frac{d\Phi_E}{dt} = i_d \quad \text{"Displacement current"}$$

Continuity

$$i = i_d$$

$$q = CV$$

$$C = \varepsilon_0 \frac{A}{d} \qquad V = Ed$$

$$q = \varepsilon_0 \frac{A}{d} Ed = \varepsilon_0 AE = \varepsilon_0 \Phi_E$$

$$i = \frac{dq}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt} = i_d$$

ELECTROMAGNETIC WAVES

Maxwell's Equations in Free Space

$$q = 0 \qquad\qquad i = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = 0 \qquad\qquad \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0 \qquad\qquad \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

ELECTROMAGNETIC WAVES

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \qquad \frac{\partial B}{\partial x} = -\varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$$

$$\frac{E_m}{B_m} = \frac{E}{B} = c$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.0 \times 10^8 \text{ m/s}$$

Maxwell's Equations

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0}$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0\left(i + \varepsilon_0 \frac{d\Phi_E}{dt}\right)$$

$$= \mu_0(i + i_d)$$