Review of Classical Physics

Developed for the Azera Group

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Classical Mechanics

- First began with Galileo (1584-1642), whose experiments with falling bodies (and bodies rolling on an incline) led to Newton's 1st Law.
- Newton (1642-1727) then developed his 3 laws of motion, together with his universal law of gravitation.
- Two additional, highly mathematical frameworks were developed by the French mathematician Lagrange (1736-1813) and the Irish mathematician Hamilton (1805-1865).
- Together, these three alternative frameworks by Newton, Lagrange, and Hamilton make up what is generally called *Classical Mechanics*.
- They are distinct from the other great forms of non-classical mechanics, *Relativistic Mechanics* and *Quantum Mechanics*, but both of these borrow heavily from *Classical Mechanics*.

Classical Physics

By the late part of the 19th century, physics consisted of two areas of research:

a) mechanics including thermodynamics

b) electromagnetism.

However, a series of problems concerning the interaction of matter with electromagnetic radiation continued to perplex physicists.

Modern physics is the ongoing study of the interactions of electromagnetics with matter within the context of quantum mechanics.

Space and Time

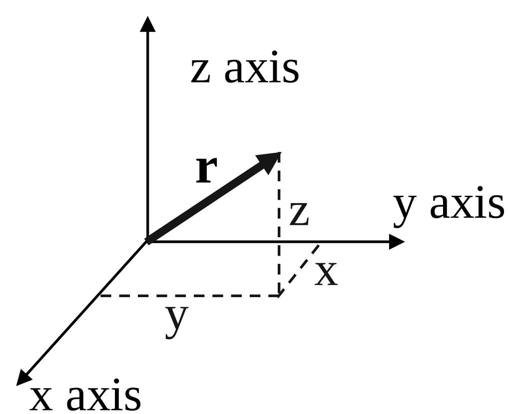
- We live in a three-dimensional world, and for all practical Engineering models, we can consider space and time to be a fixed framework against which we can make measurements of moving bodies.
- Each point *P* in space can be labeled with a distance and direction from some arbitrarily chosen origin *O*. Expressed in terms of unit vectors
- It is equivalent to write the vector as an ordered triplet of values

Ways of writing vector notation

$$\mathbf{F} = m\mathbf{a}$$

$$\vec{F} = m\vec{a}$$

$$\underline{F} = m\underline{a}$$



Vector Notations

- You may be used to the unit vector notation $\mathbf{i}, \mathbf{j}, \mathbf{k}$, but we will use the following notation: $\mathbf{x}, \mathbf{y}, \mathbf{z}$
- At times, it is more convenient to use notation that makes it easier to use summation notation, so we introduce the equivalents:

$$r_1 = x, \quad r_2 = y, \quad r_3 = z$$

$$\mathbf{e}_1 = \hat{\mathbf{x}}, \quad \mathbf{e}_2 = \hat{\mathbf{y}}, \quad \mathbf{e}_3 = \hat{\mathbf{z}}$$

which allows us to write

$$\mathbf{r} = r_1\mathbf{e}_1 + r_2\mathbf{e}_2 + r_3\mathbf{e}_3 = \sum_{i=1}^{3} r_i\mathbf{e}_i$$

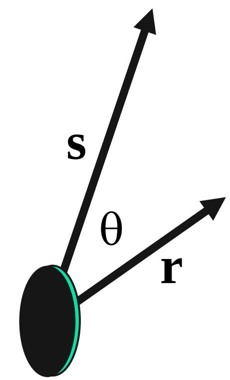
- In the above example, this form has no real advantage, but in other cases we will meet, this form is much simpler to use. The point is that we may choose any convenient notation and the physics are not changed by a change in notation.

Vector Operations

- Sum of vectors $\mathbf{r} = (r_1, r_2, r_3); \quad \mathbf{s} = (s_1, s_2, s_3) \quad \rightarrow \quad \mathbf{r} + \mathbf{s} = (r_1 + s_1, r_2 + s_2, r_3 + s_3)$
- Vector times scalar $c\mathbf{r} = (cr_1, cr_2, cr_3)$
- Scalar product, or dot product

$$\mathbf{r} \cdot \mathbf{s} = rs\cos\theta$$

$$= r_1 s_1 + r_2 s_2 + r_3 s_3 = \sum_{n=1}^{3} r_n s_n$$



- Vector product, or cross product

$$\mathbf{p} = \mathbf{r} \times \mathbf{s}; \quad |\mathbf{r} \times \mathbf{s}| = rs\sin\theta$$

$$p_x = r_y s_z - r_z s_y$$
$$p_y = r_z s_x - r_x s_z$$
$$p_z = r_x s_y - r_y s_x$$

$$\mathbf{r} \times \mathbf{s} = \det\begin{bmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ r_x & r_y & r_z \\ s_x & s_y & s_z \end{bmatrix} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ r_x & r_y & r_z \\ s_x & s_y & s_z \end{vmatrix}$$

Differentiation of Vectors

- This presentation makes use of Calculus and differential equations. In general, we will provide a brief introduction to the techniques you will need as they come up, but we will do so from an Engineering perspective—We will not be worried about the underlying mathematical proofs.
- What we need now is a simple form of something called Vector Calculus. As long as you remember that vectors are just triplets of numbers, and vector equations can be thought of as three separate equations, you will be fine.
- For now, consider only the derivative of the position vector $\mathbf{r}(t)$, which you should know gives the velocity $\mathbf{v}(t) = d\mathbf{r}(t)/dt$. Likewise, the derivative of the velocity (the second derivative of the position) gives the acceleration: $\mathbf{a}(t) = d\mathbf{v}(t)/dt = d^2\mathbf{r}(t)/dt^2$. Formally:

for scalars: $$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \quad \text{where} \quad \Delta x = x(t+\Delta t) - x(t)$$

for vectors: $$\frac{d\mathbf{r}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} \quad \text{where} \quad \Delta \mathbf{r} = \mathbf{r}(t+\Delta t) - \mathbf{r}(t)$$

Differentiation of Vectors

- By the usual rules of differentiation, the derivative of a sum of vectors is

$$\frac{d}{dt}(\mathbf{r}+\mathbf{s}) = \frac{d\mathbf{r}}{dt} + \frac{d\mathbf{s}}{dt}$$

- and the derivative of a scalar times a vector follows the usual product rule

$$\frac{d}{dt}(f\mathbf{r}) = f\,\frac{d\mathbf{r}}{dt} + \frac{df}{dt}\mathbf{r}$$

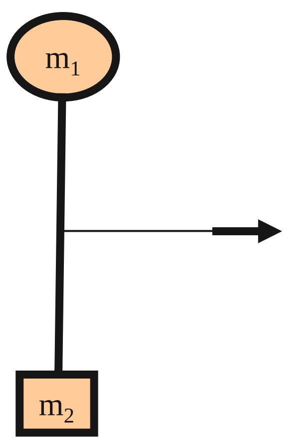
- Also note: $\mathbf{r} = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$ so $\frac{d\mathbf{r}}{dt} = \frac{dx}{dt}\hat{\mathbf{x}} + \frac{dy}{dt}\hat{\mathbf{y}} + \frac{dz}{dt}\hat{\mathbf{z}}$ $\mathbf{v} = v_x\hat{\mathbf{x}} + v_y\hat{\mathbf{y}} + v_z\hat{\mathbf{z}}$

  implies that the unit vectors are constant (i.e. $\frac{d\hat{\mathbf{x}}}{dt} = \frac{d\hat{\mathbf{y}}}{dt} = \frac{d\hat{\mathbf{z}}}{dt} = 0$).

- However, we will find in other coordinate systems the unit vectors are NOT constant!

Mass and Force

- What is the difference between mass and weight?
- Mass has to do with inertial force (ma). Weight has to do with gravitational force (mg). In the first case, the mass is "resistance to changes in motion" while in the second case it is a rather mysterious "attractive property" of matter. In fact, these two different properties of mass are identical, which is what Galileo's experiments showed (dropping masses off the leaning tower of Pisa).
- Inertial balance:



Allows measurement of inertial mass without mixing in gravitational force.

Point Particles

- For now, we want to focus on the concept of a point mass, or particle. This is an approximation, which is worthwhile to look at carefully. It basically refers to a body that can move through space but has NO internal degrees of freedom (rotation, flexure, vibrations).
- Later we will talk about bodies as collections of particles, or a continuous distribution of mass, and in considering such bodies the laws of motion are considerably more complicated.
- Despite this being an approximation, the approximation is still useful in many cases, such as for elementary particles (protons, neutrons, electrons), or even planets and stars (sometimes).

Newton's Three Laws

- Law of Inertia
  - In the absence of forces, a particle moves with constant velocity $\mathbf{v}$.
  - (An object in motion tends to remain in motion, an object at rest tends to remain at rest.)
- Force Law
  - For any particle of mass m, the net force $\mathbf{F}$ on the particle is always equal to the mass m times the particle's acceleration: $\mathbf{F} = m\mathbf{a}$.
- Conservation of Momentum Law
  - If particle 1 exerts a force $\mathbf{F}_{21}$ on particle 2, then particle 2 always exerts a reaction force $\mathbf{F}_{12}$ on particle 1 given by $\mathbf{F}_{12} = -\mathbf{F}_{21}$.
  - (For every action there is an equal and opposite reaction.)

Classical Physics

- General Ideas of Mechanics

1. A reference frame is a coordinate axis and origin used by an observer to describe the motion of an object.
2. An inertial reference frame is one in which the observer is not accelerating (i.e. one in which Newton's Laws are valid without adding fake forces to your free-body diagram)

$$\sum \vec{F}_{ext} = \frac{d\vec{p}}{dt}$$

Galilean Relativity

All inertial reference frames are equivalent! Another way of stating this principle is that **only relative motion can be detected**.

Transformation Equations

If you know what an observer in a particular reference frame observes then you can predict the observations made by an observer in any other reference frame. The equations that enable you to make these calculations are called **Transformation Equations**.

Invariance

Since the labeling of your coordinate axis and its origin location is arbitrary, the equations of physics should have the same form regardless when you rotate or translate your axis set. Equations that have this property are said to be invariant to the transformation.

It was shown that Maxwell's Equations are not invariant under a Galilean Transformation so E&M and Mechanics are not consistent.

General Ideas of E&M

1. Light is a **transverse wave**.

2. The **speed of light** depends **onl**y on **the medium** through which it travels and **not upon the observer**.

$$c = \frac{1}{\sqrt{\varepsilon\mu}}$$

3. The light propagates from the sun to the earth through the **luminiferous ether**.

4. Properties of the Ether Fluid

i) non-viscous - Earth doesn't slow down while traveling through the ether.
ii) incompressible - speed of light is very fast
iii) massless

Galilean Transformations: Time

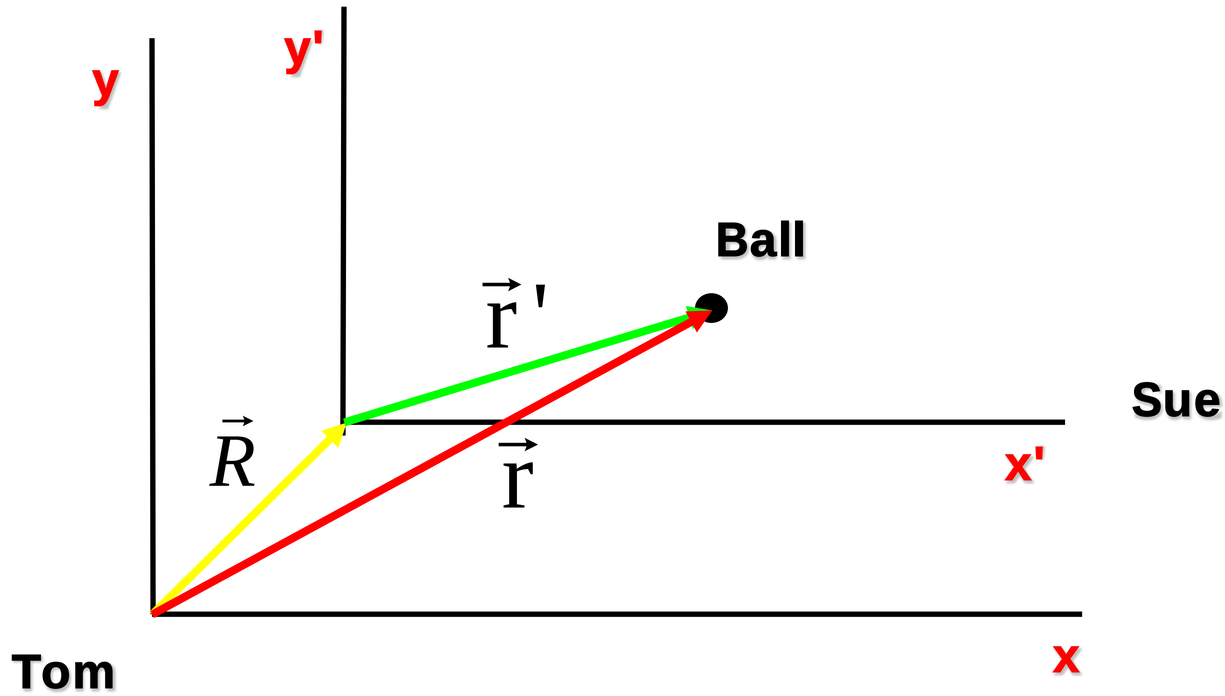
A. Time

<u>All observers measure the same time</u>. This was assumed without proof and used to derive our equations

$$\mathbf{t = t'}$$

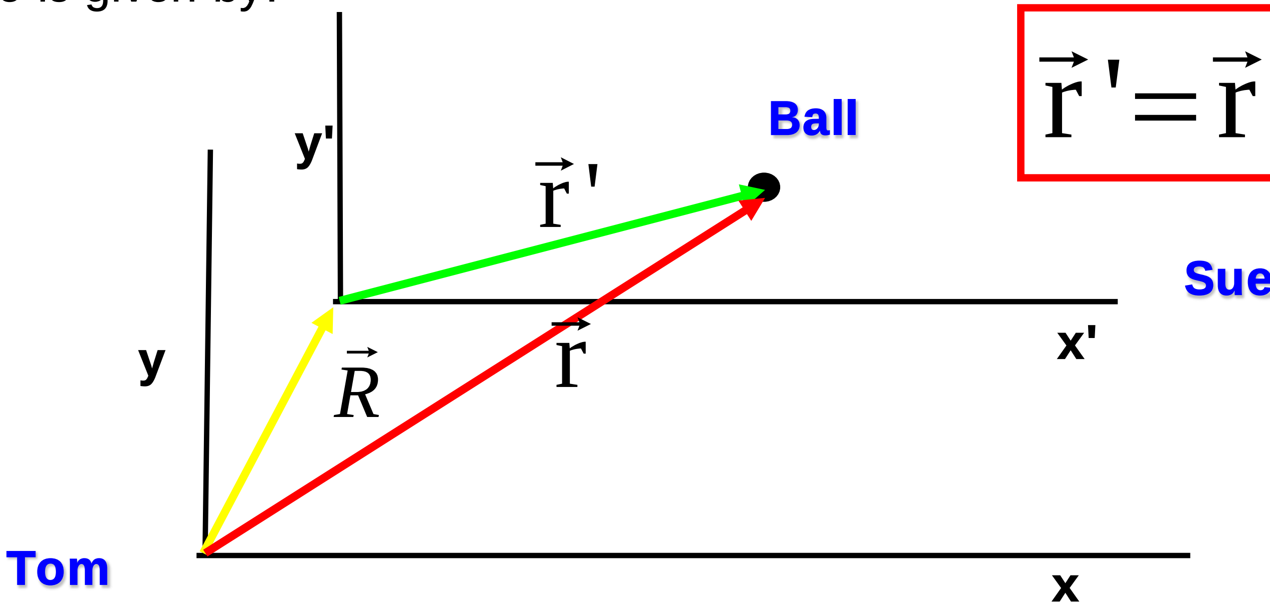
Galilean Transformations: Position

Let us consider a ball being measured by two different observers as shown below:



Galilean Transformations: Time

By vector subtraction, we see that the location of the ball according to Sue is given by:



$$\boxed{\vec{r}\,' = \vec{r} - \vec{R}}$$

Where $\vec{r}\,'$ is the position of the ball as seen by Sue

$\vec{r}$ is the position of the ball as seen by Tom

$\vec{R}$ is the position of Sue as seen by Tom

Galilean Transformations: Velocity

We now apply the time derivative operator to both sides of our position.

$$\boxed{\frac{d}{dt}\left(\vec{r}\,'\right)=\frac{d}{dt}\left(\vec{r}\right)-\frac{d}{dt}\left(\vec{R}\right)}$$

Applying the definition of velocity, we can rewrite the right-hand side of the equation as

$$\boxed{\frac{d}{dt}\left(\vec{r}\,'\right)=\vec{v}-\vec{u}}$$

where $\vec{v}$ is the velocity of the ball as seen by Tom

$\vec{u}$ is the velocity of Sue as seen by Tom

Galilean Transformations: Velocity

Since the time measured by both Sue and Tom is the same, we can replace t with t' and apply the definition of velocity to the left-hand side of the equation.

$$\boxed{\frac{d}{dt'}\left(\vec{r}\,'\right)=\vec{v}-\vec{u}} \qquad \boxed{\vec{v}\,'=\vec{v}-\vec{u}}$$

where $\vec{v}\,'$ is the velocity of the ball as seen by Sue.

You should note that the definition of velocity requires that **both** the **<u>time</u>** and **<u>position</u>** be measured by the **<u>same observer</u>**!

Galilean Transformations: Acceleration

We now follow the same procedure as in part **C** to obtain the relationship between the ball's acceleration as measured by Sue and as measured by Tom.

$$\boxed{\frac{d}{dt}\left(\vec{v}\,'\right) = \frac{d}{dt}\left(\vec{v}\right) - \frac{d}{dt}\left(\vec{u}\right)} \qquad \boxed{\frac{d}{dt}\left(\vec{v}\,'\right) = \vec{a} - \vec{A}}$$

$$\boxed{\frac{d}{dt'}\left(\vec{v}\,'\right) = \vec{a} - \vec{A}} \qquad \boxed{\vec{a}\,' = \vec{a} - \vec{A}}$$

Where $\vec{a}$ is the acceleration of the ball as seen by Tom

$\vec{a}\,'$ is the acceleration of the ball as seen by Sue

$\vec{A}$ is the acceleration of Sue as seen by Tom

Galilean Transformations: Acceleration

$$\boxed{\vec{a}' = \vec{a} - \vec{A}}$$

Where $\vec{a}$ is the acceleration of the ball as seen by Tom

$\vec{a}'$ is the acceleration of the ball as seen by Sue

$\vec{A}$ is the acceleration of Sue as seen by Tom

Note: If Sue and Tom are not accelerating with respect to each other (ie $\vec{A} = 0$), they will agree on the acceleration of the ball and Newton's Laws! The last term on the right-hand side is the reason we add fake forces when using non-inertial reference frames.

Work and Energy Concepts

A. Work

The work done by a force, , upon a body in displacing the body an amount $d\vec{s}$ is defined by the equation

$$W \equiv \int_{\text{initial}}^{\text{final}} \vec{F}\, d\vec{s}$$

B. Energy: is the ability of a body to perform work.

C. Kinetic Energy: is defined as the energy that an object has due to its motion!

Work-Energy Theorem

The work done by the net external force upon an object (or equivalently the net work done by all forces upon the object) is equal to the change in the objects kinetic energy!!

$$W_{net} \equiv \int_{initial}^{final} \left(\sum \vec{F} \right) d\vec{s} = \Delta K$$

The work-energy theorem is the heart of all energy concepts as it relates the connection between Newton II, work and energy!!

We used this theorem to derive the conservation of mechanical energy and to develop the classical formula for computing the kinetic energy of a body.

The Kinetic Energy of an Object

The kinetic energy of an object traveling at speeds much less than the speed of light (ie classical physics) can be obtained using the formula

$$K = \frac{1}{2} M v^2$$

Proof:

Inserting Newton II into the work energy theorem, we have that

$$W_{net} \equiv \int_{initial}^{final} \left(\frac{d\vec{p}}{dt} \right) d\vec{s} = \Delta K \qquad \Delta K = \int_{initial}^{final} d\vec{p} \, \frac{d\vec{s}}{dt}$$

The Kinetic Energy of an Object

We now apply the definition of velocity and linear momentum to our equation

$$\Delta K = \int_{\text{initial}}^{\text{final}} d(m\vec{v})\ \vec{v}$$

In classical mechanics, the mass of a particle is constant (an assumption we will have to re-examine in special relativity) so

$$\Delta K = \int_{\text{initial}}^{\text{final}} m\, d\vec{v}\ \vec{v}$$

We can simplify our equation by using the following Calculus

$$d(v^2) = d(\vec{v}\cdot\vec{v}) = d\vec{v}\cdot\vec{v} + \vec{v}\cdot d\vec{v} = 2\,d\vec{v}\cdot\vec{v}$$

The Kinetic Energy of an Object

Thus, our equation is simplified to

$$\Delta K = \int_{\text{initial}}^{\text{final}} \frac{1}{2} m \, d\left(v^2\right) \quad \text{and} \quad \Delta K = \frac{1}{2} m \int_{\text{initial}}^{\text{final}} d\left(v^2\right)$$

We then integrate to get: $K_{\text{final}} - K_{\text{initial}} = \frac{1}{2} m \, v^2_{\text{final}} - \frac{1}{2} m v^2_{\text{initial}}$

By comparing the individual terms, we obtain our classical formula for kinetic energy.