Three Phase Alternating Current

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Three Phase Alternating Current

Three-phase electric power is a common method of alternating current electric power generation, transmission, and distribution. It is a type of polyphase system and is the most common method used by electrical grids worldwide to transfer power.
It is also used to power large motors and other heavy loads.
A three-wire three-phase circuit is usually more economical than an equivalent two-wire single-phase circuit at the same line to ground voltage because it uses less conductor material to transmit a given amount of electrical power.

Three Phase Alternating Current

- **Introduction to Faraday's Law -One**

Faraday's law of induction (briefly, **Faraday's law**) is a basic law of electromagnetism predicting how a magnetic field will interact with an electric circuit to produce an electromotive force (EMF)—a phenomenon known as electromagnetic induction.

It is the fundamental operating principle of electrical generation facilities, transformers, inductors, and many types of electrical motors and solenoid.s

Three Phase Alternating Current

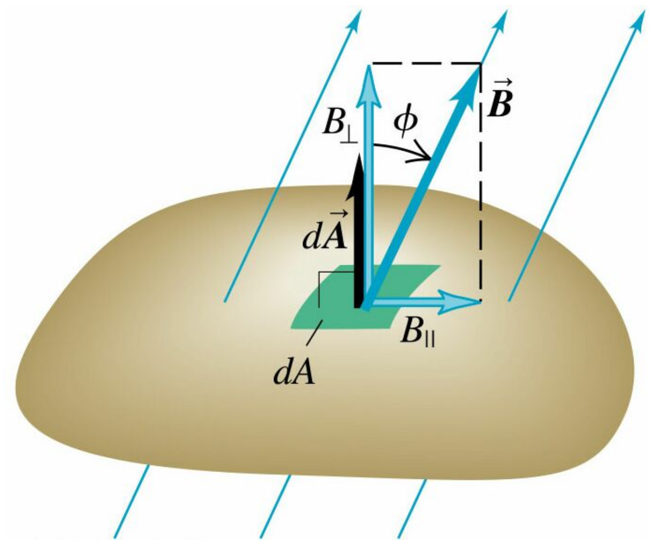
❖ Introduction to Faraday's Law-Two

We can define a magnetic flux

$$d\Phi_B = \vec{B} \cdot d\vec{A}$$

where *dA* is an incremental area with the total flux being given

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$



Note that this integral is *not* over a closed surface, for that integral would yield zero for an answer, since there are no sources or sinks for the magnetic as there is with electric fields

Three Phase Alternating Current

- **Introduction to Faraday's Law-Three**

It was Michael Faraday who was able to link the induced current with a changing magnetic flux

He stated that:

"The induced emf (electromotive force) in a closed loop equals the negative of the time rate of change of the magnetic flux through the loop"

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

The induced emf *opposes* the change that is occurring

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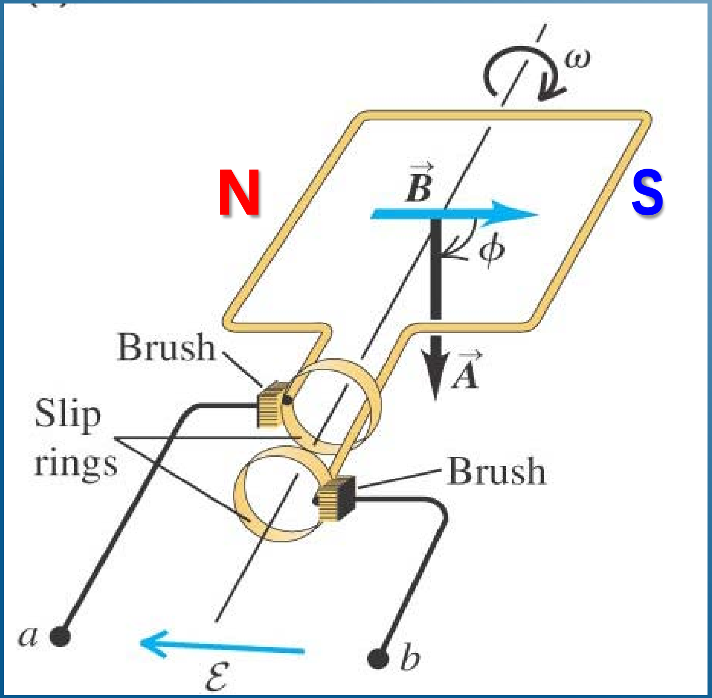
❖ **Introduction to Faraday's Law-Four**

(Faraday's Law) $$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad \text{(Volts)}$$

- **Induced emf (electro-magnetic force)** in a closed loop equals the *negative* time rate of change of magnetic flux through the loop



$$\Phi_B = \vec{B} \cdot \vec{A} = B \cdot A \cos\phi$$

for

$$\phi = \omega t$$

$$\mathcal{E} = B \cdot A \sin \omega t$$

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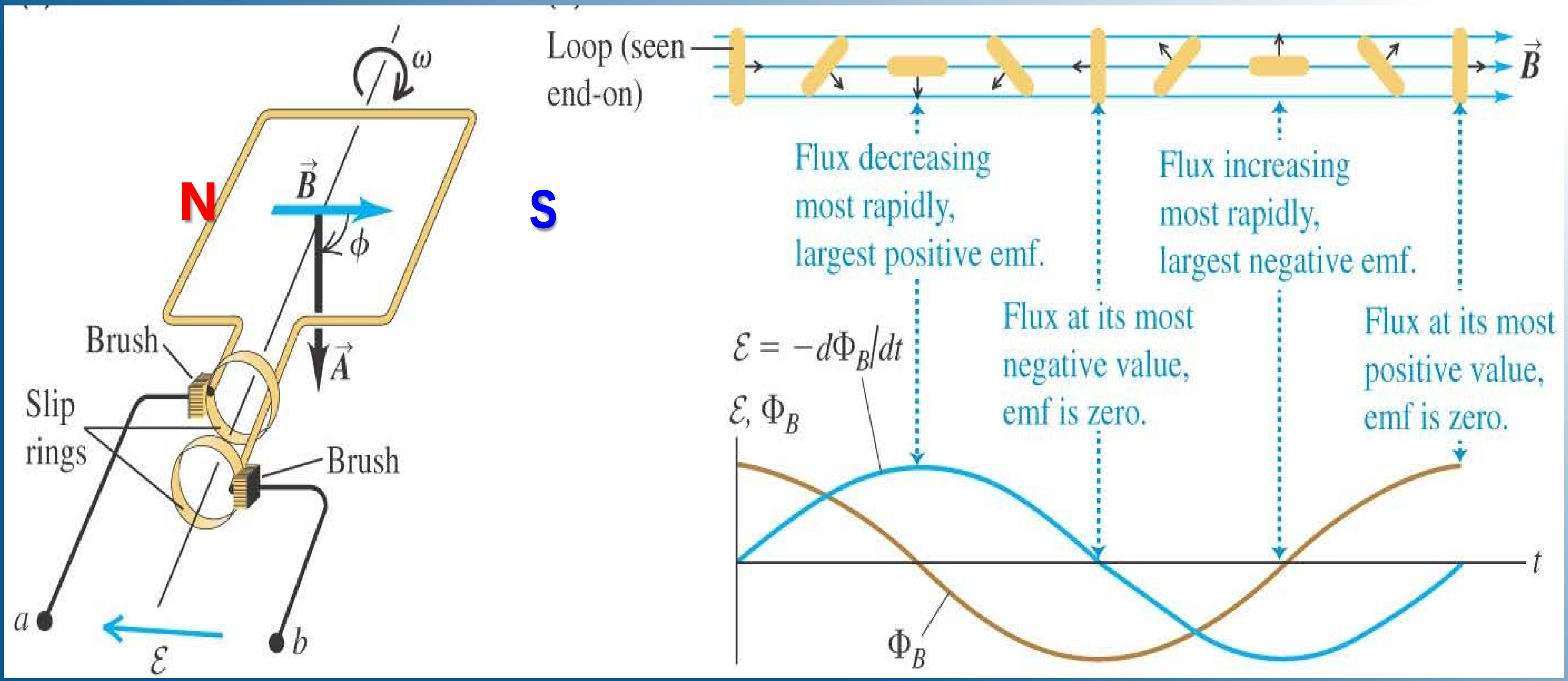
❖ Introduction to Faraday's Law-Five



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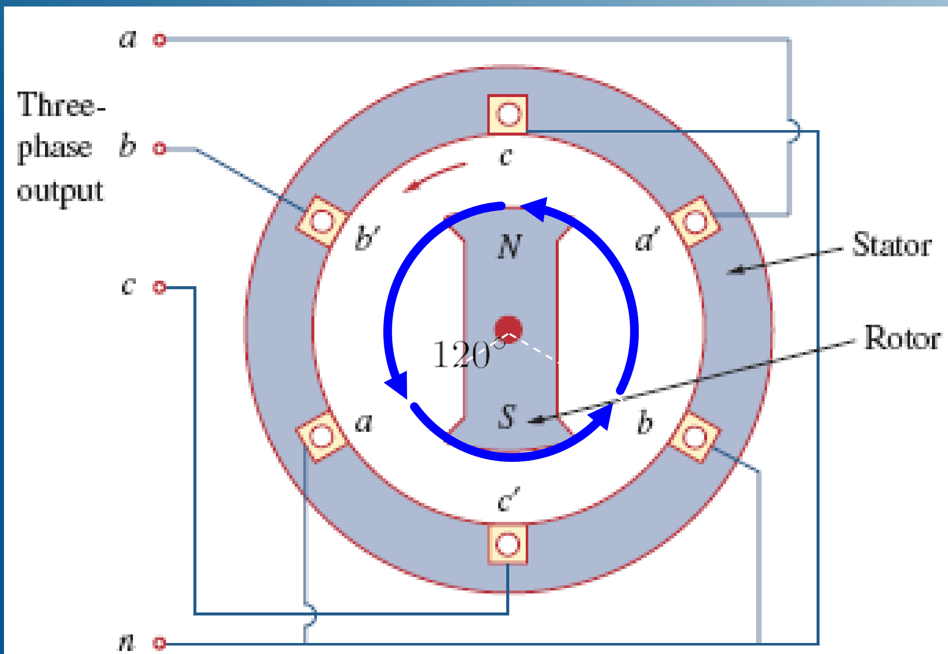
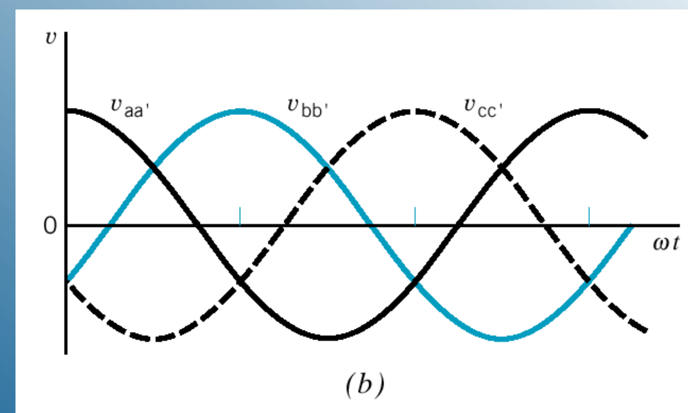
❖ Generating Three-phase Voltage

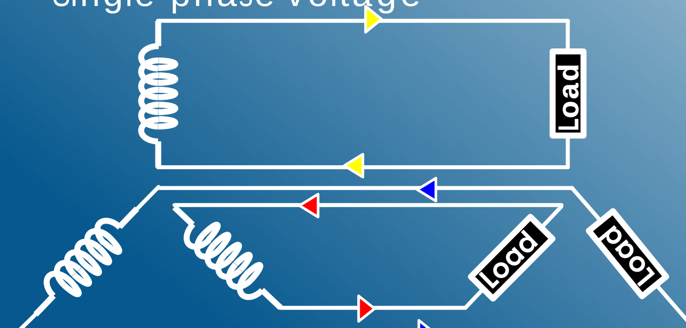


$$V_{aa'} = \sqrt{2}V\cos\omega t$$
$$V_{bb'} = \sqrt{2}V\cos(\omega t - 120^\circ)$$
$$V_{cc'} = \sqrt{2}V\cos(\omega t - 240^\circ)$$



- Three-phase voltage can be also generated with three single-phase voltage

Single-phase voltage



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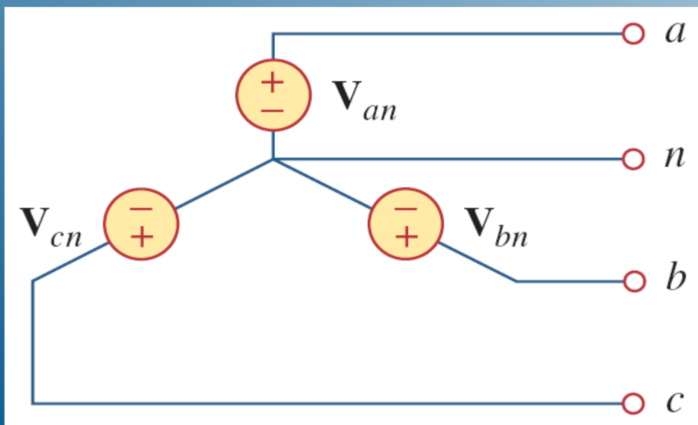
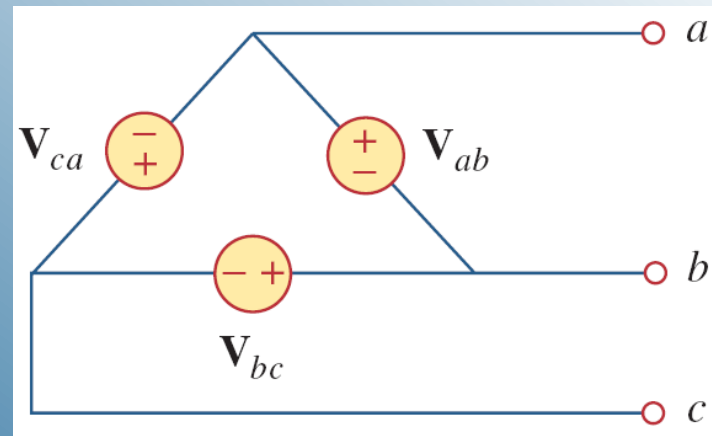
- **Generating Three-phase Voltage:**

Two Methods of Source Connections





$$\mathbf{V}_{an} = V_p \angle 0^\circ = V_p \cos(0^\circ) + V_p \sin(0^\circ) = V_p$$

$$\mathbf{V}_{bn} = V_p \angle -120^\circ = V_p \cos(-120^\circ) + V_p \sin(-120^\circ) = V_p(-0.5 - 0.866j)$$

$$\mathbf{V}_{bn} = V_p \angle -240^\circ = V_p \angle 120^\circ = V_p \cos(-240^\circ) + V_p \sin(-240^\circ) = V_p(-0.5 + 0.866j)$$

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

$\Rightarrow$ voltages between the **lines** a,b,c and the **neutral line** n: **phase voltages**.

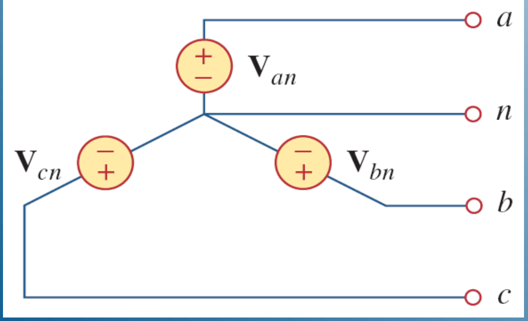
$\Rightarrow$ voltages between the **lines** V_{ab}, V_{bc}, V_{ca}: **phase voltages**.

Three Phase Alternating Current

❖ Phase Sequence

- Balanced phase voltages are equal in magnitude and are out of phase with each other by 120^0
- There are two possible ways in which a source can be balanced

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$
$$\mathbf{V}_{bn} = V_p\angle -120^\circ$$
$$\mathbf{V}_{cn} = V_p\angle -240^\circ = V_p\angle +120^\circ$$

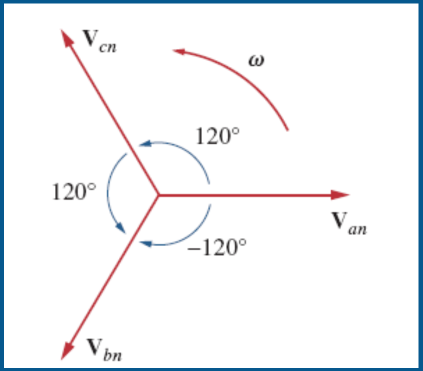


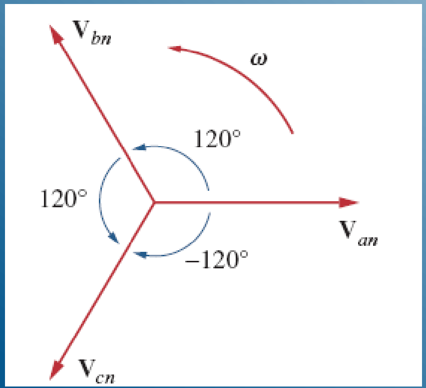
$$\mathbf{V}_{an} = V_p\angle 0^\circ$$
$$\mathbf{V}_{cn} = V_p\angle -120^\circ$$
$$\mathbf{V}_{bn} = V_p\angle -240^\circ = V_p\angle +120^\circ$$

Positive or abc sequence

The phase sequence may also be regarded as the order in which the phase voltages reach their peak (or maximum) values with respect to time.

Negative or acb sequence





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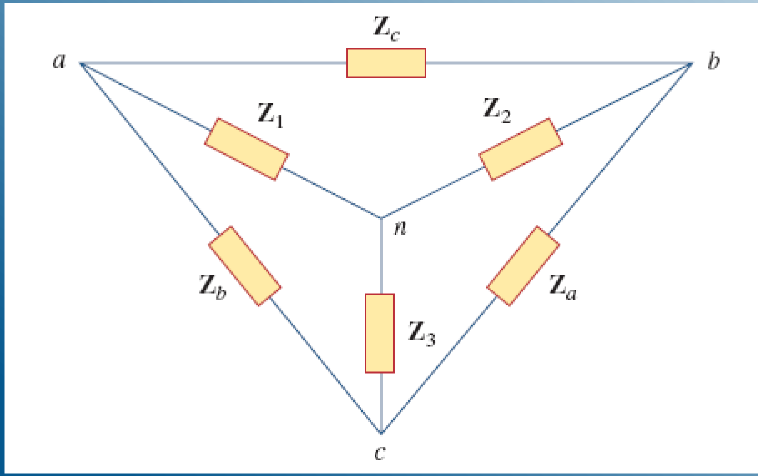
❖ Y-Delta Conversion



Y-Δ Conversion:

$$Z_a = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}$$

$$Z_b = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}$$

$$Z_c = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$

$$Z_\Delta = 3Z_Y \qquad \text{or} \qquad Z_Y = \frac{1}{3}Z_\Delta$$

Δ-Y Conversion:

$$Z_1 = \frac{Z_bZ_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_cZ_a}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_aZ_b}{Z_a + Z_b + Z_c}$$

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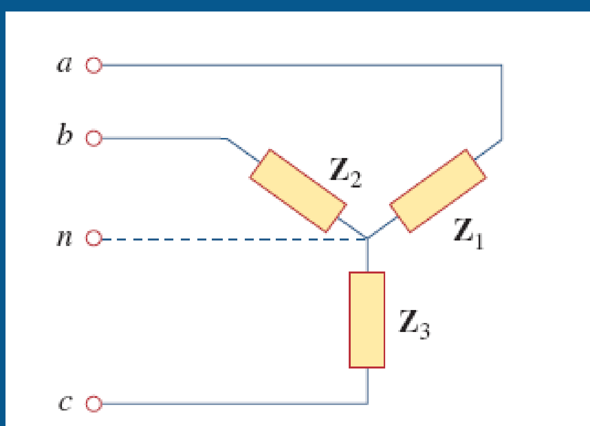
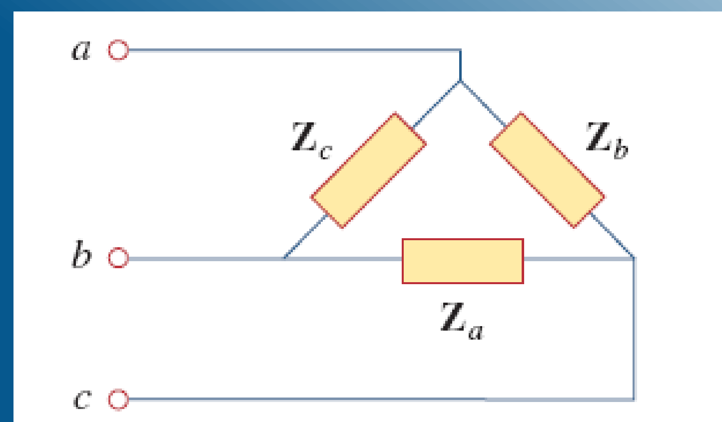
❖ Balanced Three-phase Load

- ❖ Similar to the source, the load can also be Delta or Wye connected.
- ❖ A balanced load: the phase impedances are equal in magnitude and in phase.





$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y$$

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \qquad \text{or} \qquad \mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta$$

$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_\Delta$$

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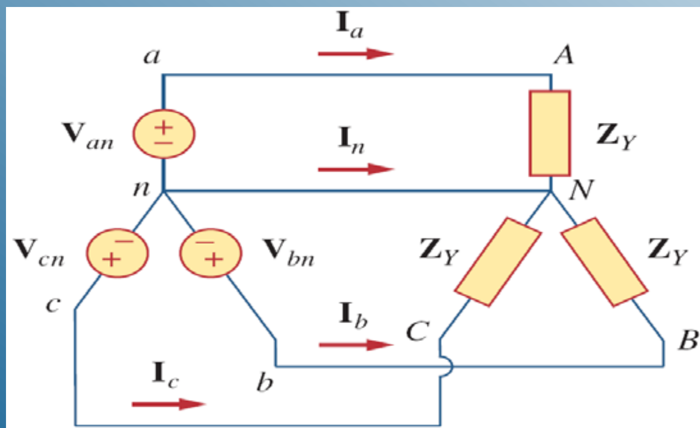
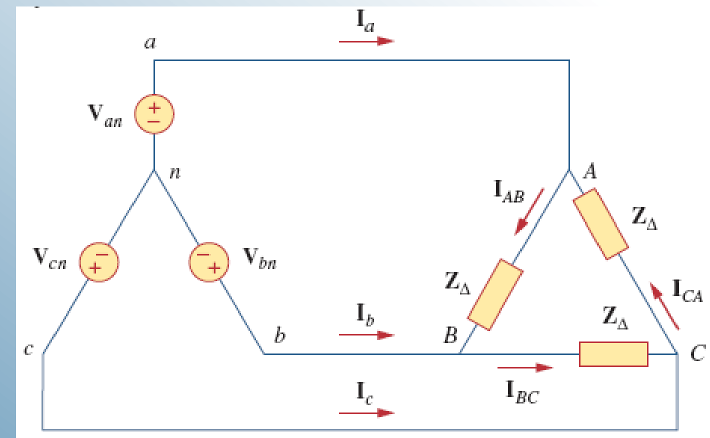
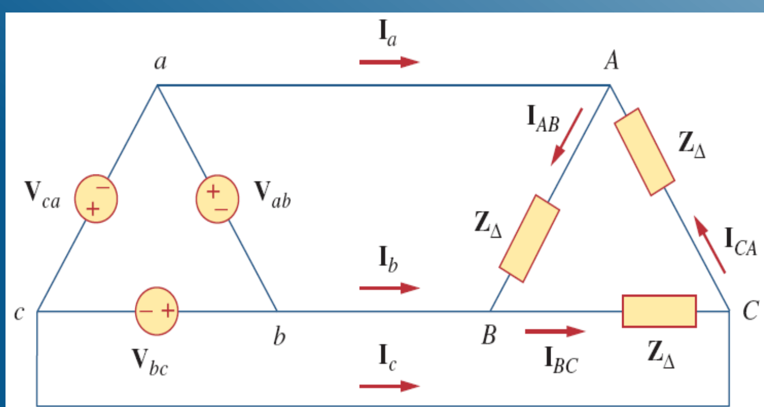
❖ Connections with Loads



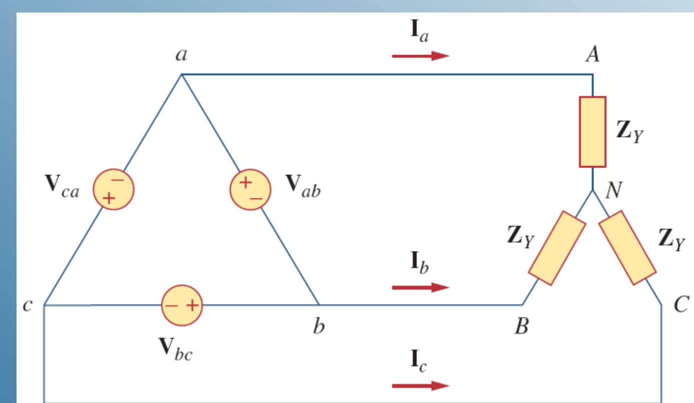
Balanced Y−Y connection



Balanced Y−Δ connection



Balanced Δ − Δ connection



Balanced Δ−Y connection

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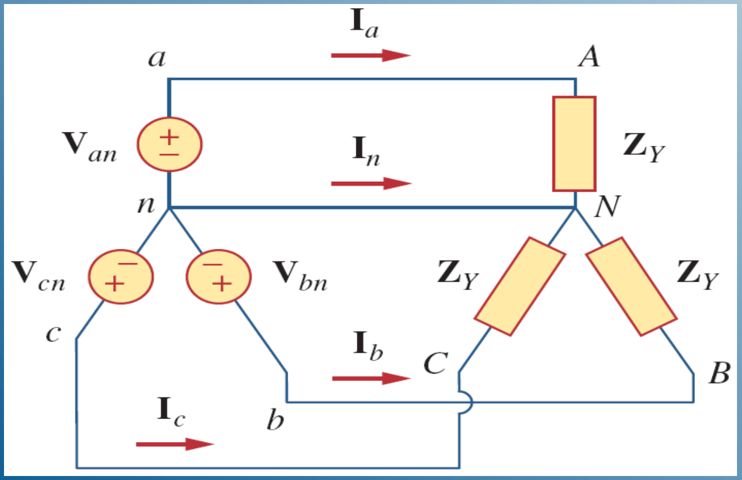
❖ Balanced Wye-Wye Connection-One



Balanced Y−Y connection

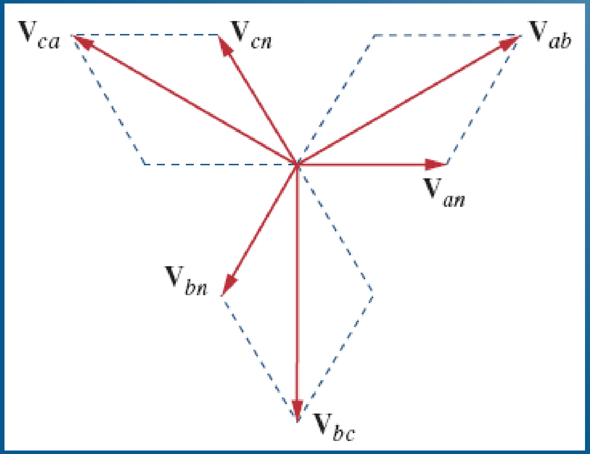
Balanced load:

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y$$

Line voltage (or line-to-line voltage)

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\angle 0^\circ - V_p\angle -120^\circ$$

$$= V_p\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}V_p\angle 30^\circ$$



$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}V_p\angle -210^\circ$$

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}V_p\angle -90^\circ$$

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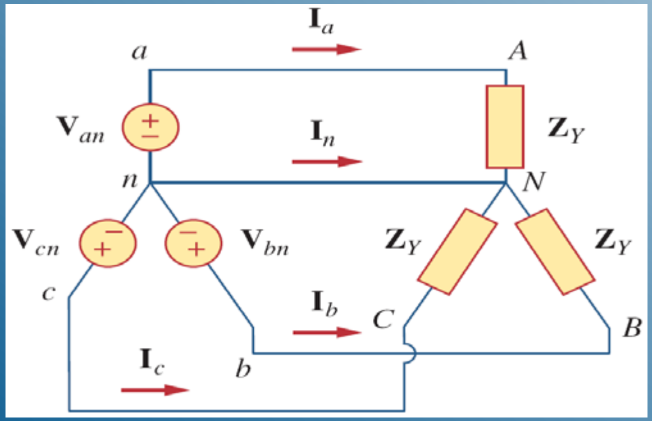
❖ Balanced Wye-Wye Connection-Two

By Kirchhov's Voltage Law (KVL), the current along the line (line current):



Balanced Y–Y connection

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

$$\mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -120^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a \angle -120^\circ$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -240^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a \angle -240^\circ$$

$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0 \qquad \mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 0 \qquad \mathbf{V}_{nN} = \mathbf{Z}_n \mathbf{I}_n = 0$$

Voltage across neutral line is zero: it can be removed

- Phase current is the current in each phase of the source of load.
- In Y-Y system, the line current is the same as the phase current.

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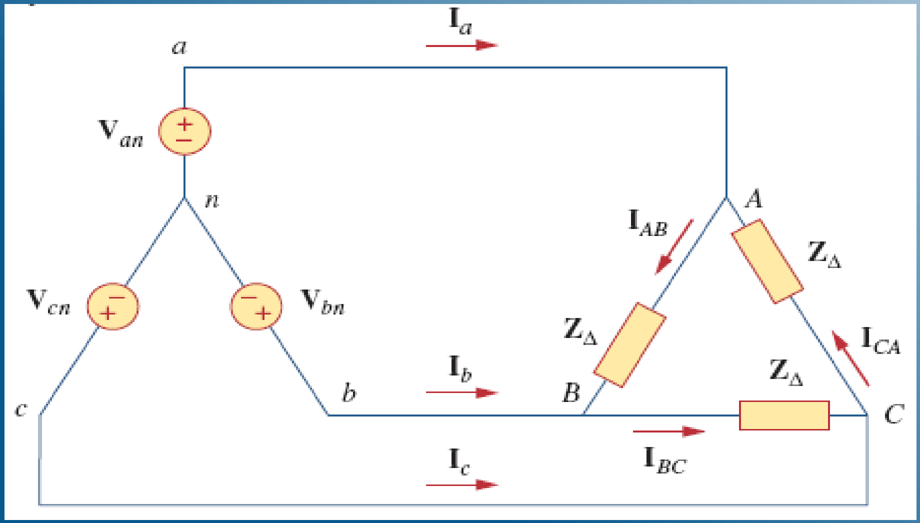
❖ Balanced Wye-Delta Connection-One



Phase voltage

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$
$$\mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ$$

Line voltage is

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn}$$
$$= \sqrt{3}V_p\angle 30^\circ = \mathbf{V}_{AB}$$
$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}V_p\angle -90^\circ = \mathbf{V}_{BC}$$
$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}V_p\angle -210^\circ$$

- Phase current can be

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta}$$

- Alternatively,

$$-\mathbf{V}_{an} + \mathbf{Z}_\Delta\mathbf{I}_{AB} + \mathbf{V}_{bn} = 0 \quad \Rightarrow \quad \mathbf{I}_{AB} = \frac{\mathbf{V}_{an} - \mathbf{V}_{bn}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}$$

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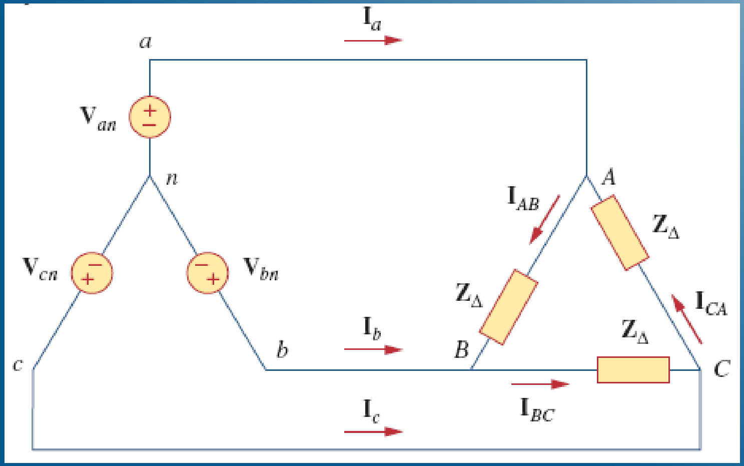
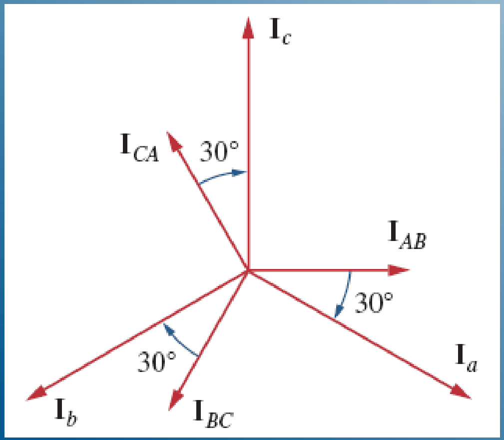
❖ Balanced Wye-Delta Connection-Two



- Line currents are obtained by KCL

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\angle{-240^\circ}) = \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\angle{-30^\circ}$$

$$\mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$



- Magnitude of line currents:

$$I_L = \sqrt{3} I_p$$

$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$

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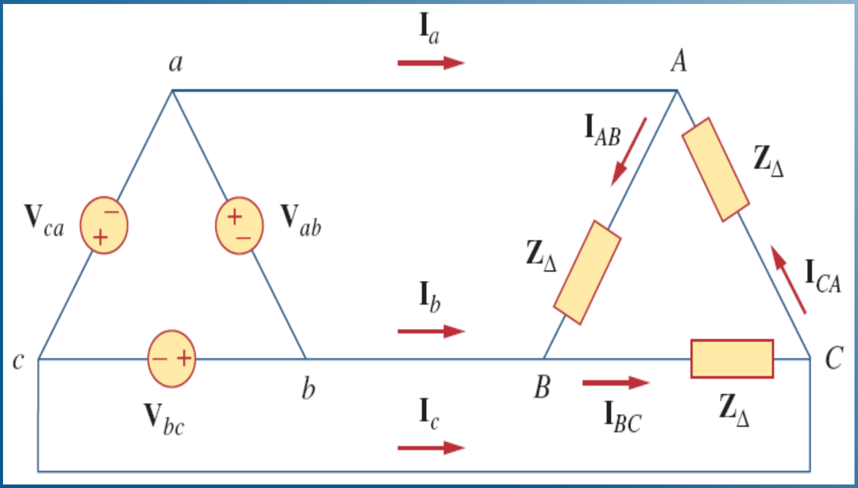
❖ Balanced Delta-Delta Connection-One



Assuming a positive sequence, the phase voltages for a delta-connected source are

$$\mathbf{V}_{ab} = V_p\angle 0^\circ \quad \mathbf{V}_{bc} = V_p\angle -120^\circ \quad \mathbf{V}_{ca} = V_p\angle +120^\circ$$

$\Rightarrow$ The line voltages are the same as the phase voltage

The phase currents $\mathbf{V}_{ab} = \mathbf{V}_{AB}, \quad \mathbf{V}_{bc} = \mathbf{V}_{BC}, \quad \mathbf{V}_{ca} = \mathbf{V}_{CA}$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_\Delta} = \frac{\mathbf{V}_{ab}}{Z_\Delta} \quad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{Z_\Delta} = \frac{\mathbf{V}_{bc}}{Z_\Delta} \quad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{Z_\Delta} = \frac{\mathbf{V}_{ca}}{Z_\Delta}$$

The line currents are obtained from the phase currents by applying KCL at nodes *A*, *B*, and *C*

$$\mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} \quad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB} \quad \mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}$$

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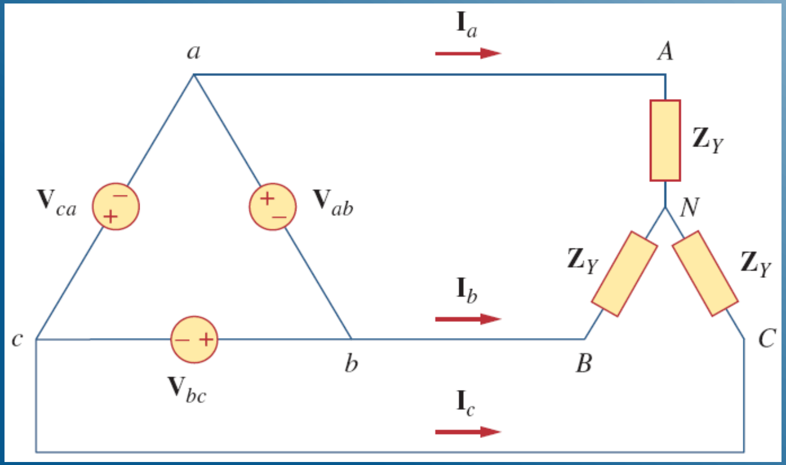
❖ Balanced Delta-Wye Connection



Assuming the *abc* sequence, the phase voltages of a delta-connected source are

These are also the line voltages as well as the phase voltages.

$$\mathbf{V}_{ab} = V_p\underline{/0^\circ} \qquad \mathbf{V}_{bc} = V_p\underline{/-120^\circ}$$

$$\mathbf{V}_{ca} = V_p\underline{/+120^\circ}$$

In order to get the line current, apply KVL to loop *aANBba*,

$$-\mathbf{V}_{ab} + \mathbf{Z}_Y\mathbf{I}_a - \mathbf{Z}_Y\mathbf{I}_b = 0 \quad \Rightarrow \quad \mathbf{Z}_Y(\mathbf{I}_a - \mathbf{I}_b) = \mathbf{V}_{ab} = V_p\underline{/0^\circ} \quad \Rightarrow \quad \mathbf{I}_a - \mathbf{I}_b = \frac{V_p\underline{/0^\circ}}{\mathbf{Z}_Y}$$

$\mathbf{I}_b$ lags $\mathbf{I}_a$ by 120°, since we assumed the *abc* sequence

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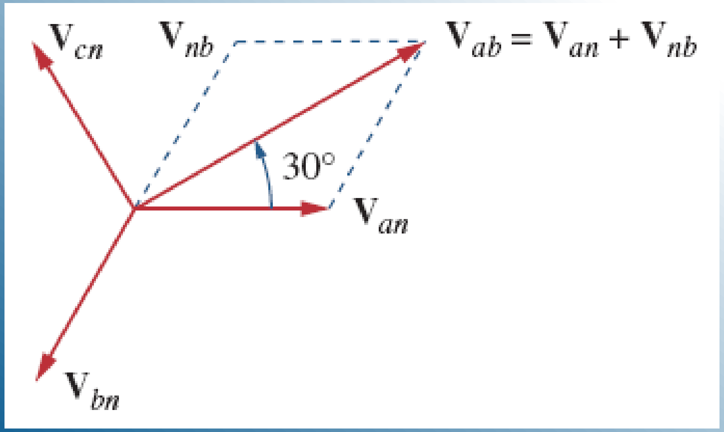
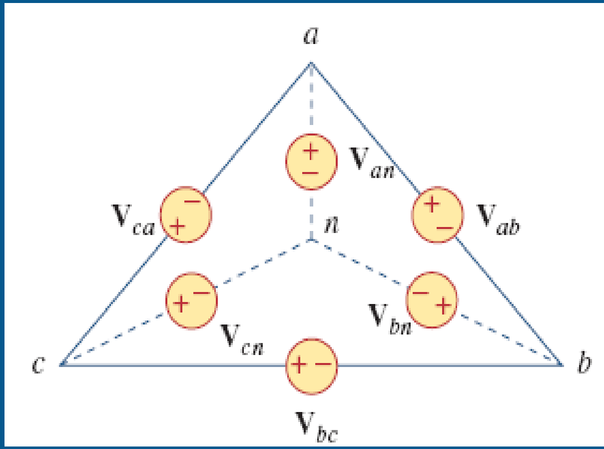
❖ Delta-Wye Transformation

- Another possible way to analyze the Delta-Star connection.
  - Transform Delta connected source to Star connected source
  - Analyze the Y-Y connection.
- Observing the phase voltage to line voltage relation in the Y-Y connection





$$\mathbf{V}_{ab} = V_p\angle 0^\circ, \qquad \mathbf{V}_{bc} = V_p\angle -120^\circ$$
$$\mathbf{V}_{ca} = V_p\angle +120^\circ$$

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle -30^\circ$$
$$\mathbf{V}_{bn} = \frac{V_p}{\sqrt{3}}\angle -150^\circ, \qquad \mathbf{V}_{cn} = \frac{V_p}{\sqrt{3}}\angle +90^\circ$$

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❖ Power in a Balanced System-One

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For a Y-connected load, the phase voltages are

(V_p (rms) of the phase voltage)

$$v_{AN} = \sqrt{2}V_p \cos \omega t, \quad v_{BN} = \sqrt{2}V_p \cos(\omega t - 120°)$$

$$v_{CN} = \sqrt{2}V_p \cos(\omega t + 120°)$$

If $\mathbf{Z}_Y = Z\angle\theta$,

the phase currents lag behind their corresponding phase voltages by θ

$$i_a = \sqrt{2}I_p \cos(\omega t - \theta), \quad i_b = \sqrt{2}I_p \cos(\omega t - \theta - 120°)$$

$$i_c = \sqrt{2}I_p \cos(\omega t - \theta + 120°)$$

The total instantaneous power in the load is the sum of the instantaneous powers in the three phases

$$\begin{aligned} p &= p_a + p_b + p_c = v_{AN}i_a + v_{BN}i_b + v_{CN}i_c \\ &= 2V_pI_p[\cos \omega t \cos(\omega t - \theta) + \cos(\omega t - 120°)\cos(\omega t - \theta - 120°) + \cos(\omega t + 120°)\cos(\omega t - \theta + 120°)] \end{aligned}$$

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❖ Power in a Balanced System-Two

Applying the trigonometric identity,

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

$$\begin{aligned} p &= V_p I_p[3 \cos \theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240°) \\ &\qquad + \cos(2\omega t - \theta + 240°)] \\ &= V_p I_p[3 \cos \theta + \cos \alpha + \cos \alpha \cos 240° + \sin \alpha \sin 240° \\ &\qquad + \cos \alpha \cos 240° - \sin \alpha \sin 240°] \\ &\qquad \text{where } \alpha = 2\omega t - \theta \\ &= V_p I_p\left[3 \cos \theta + \cos \alpha + 2\left(-\frac{1}{2}\right)\cos \alpha\right] = 3V_p I_p \cos \theta \end{aligned}$$

Thus the total instantaneous power in a balanced three-phase system is constant—it does not change with time as the instantaneous power of each phase does.

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❖ Power in a Balanced System-Three

Average power per phase for either the Δ-connected load or the Y-connected load is $p/3$, or

$$P_p = V_p I_p \cos\theta$$

The reactive power per phase

$$Q_p = V_p I_p \sin\theta$$

The apparent power per phase

$$S_p = V_p I_p$$

The complex power per phase

$$\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p \mathbf{I}_p^*$$

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❖ Power in a Balanced System-Four

The total average power is the sum of the average powers in the phases

$$P = P_a + P_b + P_c = 3P_p = 3V_pI_p\cos\theta = \sqrt{3}V_LI_L\cos\theta$$

The total reactive power

$$Q = 3V_pI_p\sin\theta = 3Q_p = \sqrt{3}V_LI_L\sin\theta$$

The total complex power

$$\mathbf{S} = 3\mathbf{S}_p = 3\mathbf{V}_p\mathbf{I}_p^* = 3I_p^2\mathbf{Z}_p = \frac{3V_p^2}{\mathbf{Z}_p^*}$$

where $\mathbf{Z}_p = Z_p/\underline{\theta}$ is the load impedance per phase.

$$\mathbf{S} = P + jQ = \sqrt{3}V_LI_L/\underline{\theta}$$

Three Phase Alternating Current

❖ Advantage of three-phase Circuit

- Consider an amount of power P_L being transmitted at the same line voltage V_L using
  - Single phase supply
  - 3-phase balanced supply
- Power dissipation in transmission

$$I_L = \frac{P_L}{V_L}$$
$$P_{loss} = 2I_L^2 R = 2R\frac{P_L^2}{V_L^2}$$

$$I_L' = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c| = P_L/(\sqrt{3}V_L)$$
$$P_{loss}' = 3(I_L')^2 R' = 3R'\frac{P_L^2}{3V_L^2} = R'\frac{P_L^2}{V_L^2}$$

$$I_L' = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c| = P_L/(\sqrt{3}V_L)$$
$$P_{loss}' = 3(I_L')^2 R' = 3R'\frac{P_L^2}{3V_L^2} = R'\frac{P_L^2}{V_L^2}$$